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Asymptotic Behavior of Nonlinear Parabolic Partial Functional Differential Equations *

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Abstract: This paper is devoted to the investigation of the asymptotic behavior for a class of nonlinear parabolic partial functional differential equations. The boundedness and stability of the solutions are established by the upper-lower solution method. Some conditions are obtained by using the semigroup theory, the properties of nonnegative matrices and the techniques of inequalities to determine the asymptotically stable region of the equilibrium.

Key words: stable region; boundedness; upper-lower solution.

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1. Introduction

In this paper, we consider the following parabolic partial functional differential equations with Neumann boundary condition

$$\begin{cases} \frac{\partial u_i(t,\mathbf{x})}{\partial t} = \Delta u_i(t,\mathbf{x}) - \omega_i u_i(t,\mathbf{x}) + f_i(\mathbf{x},\mathbf{u},\mathbf{u_t}) \\ \qquad\qquad\qquad (i=1,\cdots,m) \quad \text{in } D \triangleq [0,+\infty)\times\Omega, \\ \frac{\partial u_i(t,\mathbf{x})}{\partial \nu} = 0 \qquad (i=1,\cdots,m) \quad \text{on } S \triangleq (0,+\infty)\times\partial\Omega, \\ u_i(t,\mathbf{x}) = \varphi_i(t,\mathbf{x}) \quad (i=1,\cdots,m) \quad \text{in } J_i\times\Omega \triangleq [-r_i,0]\times\Omega, \end{cases} \tag{1.1}$$

in which $\mathbf{x}=(x_1,\cdots,x_n)$ is a row vector, $\Delta=\sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$, ω_i are positive real number, Ω is a bounded set in $\mathbf{R}^n$ with $C^{1+\alpha}$ smooth boundary $\partial\Omega$, $\mathbf{u}=(u_1(t,\mathbf{x}),\cdots,u_m(t,\mathbf{x}))$ and $\mathbf{u_t}=(u_{1,t},\cdots,u_{m,t})$ where, for any $t\geq 0$, each $u_{i,t}$ represents a function on $J_i\times\Omega$

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defined by $u_{i,t}(\theta,\mathbf{x}) = u_i(t+\theta,\mathbf{x})$ with $\mathbf{x} \in \Omega$ and $\theta \in [-r_i, 0]$, r_i is positive real number, $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative to $\partial\Omega$ and each φ_i is a given Hőlder continuous function on $J_i \times \Omega$, $f_i(\mathbf{x},\mathbf{u},\mathbf{v})$ is Hőlder continuous in $\mathbf{x}$ with $f_i(\mathbf{x},0,0) = 0$ and a certain type of condition to be made precise later.

The asymptotic behavior of solutions for the partial functional differential equations has been studied in recent years by many authors and various results have reported (see [1]–[8]). Most of these results are concerning the global stability of partial functional differential equations (see [1]–[4]). However, there are more equilibria for nonlinear differential and global stability usually does not exist. How far can the the initial values be allowed to vary without disrupting the stability properties established in the immediate vicinity of equilibrium states? Martin and Smith[5] and Pozio[6] proved that all solutions of the equations with initial data belonging to specified region, converge to some steady state of the equations as $t \to +\infty$ uniformly in $x \in \overline{\Omega}$ under suitable hypotheses. But they didn't give out the range of the region. To our knowledge, little has been reported for results on range of the region for partial functional differential equations. In the paper, we will discuss the locally asymptotic stability of the equations (1.1) and give out some conditions to determe asymptotically stable region of the equations.

2. Notations and terminologies

Let $C(X,Y)$ denote the class of continuous mapping from the topological space X to the topological space Y and $C^\alpha(J_i \times \Omega)$ the space of Hőlder continuous functions in $J_i \times \Omega$ with exponent $\alpha \in (0,1)$. We use the product spaces $C_{\mathbf{r}}^\alpha \equiv C^\alpha(J_1 \times \Omega) \times \cdots \times C^\alpha(J_m \times \Omega)$ with $\mathbf{r} \equiv (r_1, \cdots, r_m)$ and $C(Q) \equiv C(Q^{(1)}, \mathbf{R}) \times \cdots \times C(Q^{(m)}, \mathbf{R})$ with $Q^{(i)} \triangleq [-r_i, +\infty) \times \overline{\Omega}$. The symbol $\rho(\mathbf{A})$ denotes the spectral radius of a square matrix $\mathbf{A}$. The notation $W_\rho(\mathbf{A})$ is used to denote the characteristic space associated with $\rho(\mathbf{A})$. Definitions of upper-lower of the equations (1.1) and mixed quasimonotone functions can be found in [2].

Given a pair of coupled functions ,we define a sector in $C(Q)$ by

$$\langle \hat{\mathbf{u}}, \tilde{\mathbf{u}} \rangle \equiv \{\mathbf{u} \in C(Q) : \hat{\mathbf{u}} \le \mathbf{u} \le \tilde{\mathbf{u}} \ \text{ in } Q\}.$$

Throughout this paper, we assume that $\mathbf{f}$ satisfies the locally Lipschitz continuous property and

$$\text{(A)} \qquad |f_i(\mathbf{x},\mathbf{u},\mathbf{u_t})| \le \sum_{j=1}^{m} p_{ij}([\mathbf{u_t}]^+_{\infty,\mathbf{r}}) |u_{j,t}|_{r_j}, \quad i = 1, \cdots, m, \ \ (t,\mathbf{x}) \in \overline{D}, \tag{2.1}$$

where $[\mathbf{u_t}]^+_{\infty,\mathbf{r}} \equiv (\|u_{1,t}\|_{\infty,r_1}, \cdots, \|u_{m,t}\|_{\infty,r_m})$, $\|u_{i,t}\|_{\infty,r_i} \equiv \max\limits_{-r_i \le \theta \le 0} \max\limits_{\mathbf{x} \in \overline{\Omega}} |u_i(t+\theta,\mathbf{x})|$, $|u_{i,t}|_{r_i} \equiv \max\limits_{-r_i \le \theta \le 0} |u_i(t+\theta,\mathbf{x})|$, $p_{ij}(\mathbf{v})$, $j = 1, \cdots, m$, are nonnegative nondecreasing continuous function in $\mathbf{v} \in \mathbf{R}^m_+$.

With the help of [2, Theorem 3.1], we obtain easily the following lemmas.

Lemma 2.1 *If there exists a nonnegative row vector* $\mathbf{K}$ *such that*

$$\mathbf{P}(\mathbf{K})\mathbf{K}^T \le \mathbf{K}^T, \tag{2.2}$$

where $\mathbf{P}(\mathbf{K}) = [\frac{p_{ij}(\mathbf{K})}{\omega_i}]$ *is a* $m \times m$ *matrix and* $\mathbf{K}^T$ *is transpose of* $\mathbf{K}$, *and* $\mathbf{f}$ *is mixed quasi-monotone in* $\langle -\mathbf{K}, \mathbf{K}\rangle$, *then the equations (1.1) has a unique solution* $\mathbf{u}(t,\mathbf{x}) \in \langle -\mathbf{K}, \mathbf{K}\rangle$ *when* $-\mathbf{K} \leq \varphi \leq \mathbf{K}$.

Lemma 2.2 *Under conditions of Lemma 2.1, the zero solution* $\mathbf{0}$ *of the equations (1.1) is stable.*

Definition 2.1 *A nontrivial set* $W(\mathbf{0}) \subset C^{\alpha}_{\mathbf{r}}$ *is called to be an asymptotically stable region of the equilibrium* $\mathbf{0}$ *to the equations (1.1) if the solution* $\mathbf{0}$ *of the equations is stable and for any* $\varphi \in W(\mathbf{0})$,

$$\lim_{t\to+\infty} \mathbf{u}(t,\mathbf{x}) = 0 \quad \text{uniformly for} \quad \mathbf{x} \in \overline{\Omega}. \tag{2.3}$$

When $W(\mathbf{0}) = C^{\alpha}_{\mathbf{r}}$, *the equilibrium* $\mathbf{0}$ *of the equations (1.1) is called to be globally asymptotically stable.*

The usual norms in the spaces $X_p = L^p(\Omega;\mathbf{R})$ X_∞=(or $L^\infty(\Omega;\mathbf{R})$ are denoted by

$$\|u\|_p = \{\frac{1}{|\Omega|}\int_\Omega |u(\mathbf{x})|^p \mathrm{d}\mathbf{x}\}^{\frac{1}{p}}, \quad \text{for } u \in X_p,\ 1 \leq p < +\infty,$$

$$\|u\|_\infty = \max_{\mathbf{x}\in\overline{\Omega}} |u(\mathbf{x})|, \quad \text{for } u \in X_\infty,$$

where $|\Omega|$ is measure of Ω. Define a closed linear operator $A^{(i)}_p$ in X_p with domain $D(A^{(i)}_p)$ by $A^{(i)}_p \equiv -\Delta + \omega_i$, $D(A^{(i)}_p) = \{u_i \in W^{2,p}(\Omega;\mathbf{R});\ \frac{\partial u_i}{\partial \nu} = 0 \text{ on } \partial\Omega\}$. It is well known that $-A^{(i)}_p$ generates an analytic semi-group of bounded linear operators $\{T^{(i)}_p(t)\}_{t\geq 0}$ on X_p. By elementary Hilbert space arguments (see [9]),

$$\|T^{(i)}_2(t)u_i\|_2 \leq e^{-\omega_i t}\|u_i\|_2.$$

3. Main results

In this section we will be concerned with establishing some conditions for calculating out asymptotically stable region of the equilibrium $\mathbf{0}$ for the nonlinear parabolic partial functional differential equations. The equations in (1.1) satisfy the following abstract integral equations (see [3] or [8])

$$\begin{cases} u_i(t) = T^{(i)}_p(t)\varphi_i(0,\cdot) + \int_0^t T^{(i)}_p(t-s) f_i(\mathbf{x},\mathbf{u},\mathbf{u}_s)\mathrm{d}s, \quad t \geq 0, \\ u_{i,0} = \varphi_i, \end{cases} \tag{3.1}$$

where $i = 1,\cdots,m$.

Theorem 3.1 *If* $\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{v})$ *is mixed quasimonotone for* $\mathbf{u},\mathbf{v}$ *in* $C(Q)$ *and the set* $W(\mathbf{0})\backslash\{\mathbf{0}\}$ *is nonempty, where*

$$W(\mathbf{0}) = \cup_{\mathbf{K}\in\pounds} P_{\mathbf{K}}, \quad \pounds = \{\mathbf{K} \in \mathbf{R}^m_+ : P(\mathbf{K})\mathbf{K}^T \leq \mathbf{K}^T\},$$

$$P_{\mathbf{K}} = \{\varphi \in C_{\mathbf{r}}^{\alpha} : -\mathbf{K} \leq \varphi \leq \mathbf{K}, \rho(P(\mathbf{K})) < 1\},$$

then $W(\mathbf{0})$ is an asymptotically stable region of the equilibrium $\mathbf{0}$ to the problem (1.1).

Proof Letting $\mathbf{u}(t,\mathbf{x}) \equiv \mathbf{u}(t)(\mathbf{x})$ be a solution of (1.1), we have, from (3.1), taking $p = 2$,

$$\begin{aligned}\|u_i(t)\|_2 &\leq \|T_2^{(i)}(t)\varphi_i(0,\cdot)\|_2 + \int_0^t \|T_2^{(i)}(t-s)f_i(\cdot,\mathbf{u},\mathbf{u_s})\|_2 \mathrm{d}s,\\ &\leq e^{-\omega_i t}\|\varphi_i(0,\cdot)\|_2 + \int_0^t e^{-\omega_i(t-s)}\{\frac{1}{|\Omega|}\int_\Omega [\sum_{j=1}^m p_{ij}([\mathbf{u_s}]^+_{\infty,\mathbf{r}})|u_{j,s}|_{r_j}]^2 \mathrm{d}x\}^{\frac{1}{2}} \mathrm{d}s,\\ &\qquad t \geq 0. \end{aligned} \tag{3.2}$$

Stability of the solution $\mathbf{0}$ is obvious by Lemma 2.1, we only prove that (2.3) holds.

We first show that, for $\varphi \in W(\mathbf{0})$, that is, there is a nonnegative row vector $\mathbf{K}$ such that $\mathbf{P}(\mathbf{K})\mathbf{K}^T \leq \mathbf{K}^T$, $-\mathbf{K} \leq \varphi \leq \mathbf{K}$, and $\rho(\mathbf{P}(\mathbf{K})) < 1$, we have,

$$\lim_{t\to+\infty} \|u_i(t)\|_2 = 0. \tag{3.3}$$

By Lemma 2.1, it follows that $\mathbf{u} \in \langle -\mathbf{K}, \mathbf{K}\rangle$ when $-\mathbf{K} \leq \varphi \leq \mathbf{K}$. Thus $\|u_i(t)\|_2 \leq K_i$, for $i = 1,\cdots,m$, where K_i is i-th component of vector $\mathbf{K}$. So there is a nonnegative constant row vector $\sigma = (\sigma_1,\cdots,\sigma_m)$ such that

$$\lim_{t\to+\infty} \sup \|u_i\|_2 = \sigma_i, \quad i = 1,\cdots,m. \tag{3.4}$$

According to the definition of limsup and (3.4), for a sufficient small constant $\varepsilon > 0$, there is $t_1 > 0$ such that, for any $t \geq t_1$,

$$\|u_i(t+\theta,\cdot)\|_2 \leq (1+\varepsilon)\sigma_i, \ \ i = 1,\cdots,m, \ \ -r_i \leq \theta \leq 0. \tag{3.5}$$

Taking $T > \max_{1\leq i\leq m}\{-\frac{\ln\varepsilon}{\omega_i}\}$, for the above ε, there must be

$$\int_T^{+\infty} e^{-\omega_i s}\mathrm{d}s < \varepsilon. \tag{3.6}$$

Then, by the boundedness of $\mathbf{u}$, (3.2), (3.5) and (3.6), we get, for $t > t_1 + T$,

$$\begin{aligned}\|u_i(t)\|_2 \leq & e^{-\omega_i t}\|\varphi_i\|_{2,r_i} + \int_0^{t-T} e^{-\omega_i(t-s)}\sum_{j=1}^m p_{ij}(\mathbf{K})\|u_{j,s}\|_{2,r_j}\mathrm{d}s +\\ & \int_{t-T}^t e^{-\omega_i(t-s)}\sum_{j=1}^m p_{ij}(\mathbf{K})(1+\varepsilon)\sigma_j \mathrm{d}s\\ \leq & e^{-\omega_i t}\|\varphi_i\|_{2,r_i} + \varepsilon\sum_{j=1}^m p_{ij}(\mathbf{K})K_j + \frac{1-e^{-\omega_i T}}{\omega_i}\sum_{j=1}^m p_{ij}(\mathbf{K})(1+\varepsilon)\sigma_j,\end{aligned}$$

where $\|\varphi_i\|_{2,r_i} \equiv \max_{-r_i\leq\theta\leq 0}\|\varphi_i\|_2$.

Thus, letting $t \to +\infty$ and $\varepsilon \to 0$, we get

$$\sigma_i \le \frac{\sum_{j=1}^{m} p_{ij}(\mathbf{K})\sigma_j}{\omega_i} \quad \text{or} \quad \sigma^T \le \mathbf{P}(\mathbf{K})\sigma^T.$$

If $\sigma \ge 0$ and $\sigma \ne 0$, then, by Theorem 8.3.2 of [10],

$$\rho(\mathbf{P}(\mathbf{K})) \ge 1.$$

This contradicts $\rho(\mathbf{P}(\mathbf{K})) < 1$. Hence (3.3) holds.

Next, we prove that

$$\lim_{t\to+\infty} u_i(t,\mathbf{x}) = 0 \quad i = 1,\cdots,m, \quad \text{uniformly for} \quad \mathbf{x} \in \overline{\Omega}.$$

By means of the boundedness of $u_i(t,\mathbf{x})$ and the following inequality, for any $p \ge 2$,

$$\|u_i(t,\mathbf{x})\|_p \le \|u_i(t,\mathbf{x})\|_\infty^{\frac{p-2}{p}} \|u_i(t,\mathbf{x})\|_2^{2/p}, \quad i = 1,\cdots,m, \ t \ge 0,$$

we have

$$\|u_i(t,\mathbf{x})\|_p \le M, \quad i = 1,\cdots,m, \ t \ge 0,$$

in which M is a positive constant and $\lim_{t\to+\infty} \|u_i(t,\mathbf{x})\|_p = 0 \quad i = 1,\cdots,m$. By using the same argument similar to that of Yamada[8], we obtain (2.3), and the proof is complete. □

Theorem 3.2 *If $\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{v})$ is mixed quasimonotone for $\mathbf{u},\mathbf{v}$ in $C(Q)$ and the set $W^e(0)\backslash\{0\}$ is nonempty, where*

$$W^e(0) = \cup_{\mathbf{K}\in\mathcal{L}} P_{\mathbf{K}}, \quad \mathcal{L} = \{\mathbf{K} \in \mathbf{R}_+^{\mathbf{m}} : P(\mathbf{K})\mathbf{K}^T \le \mathbf{K}^T\},$$

$$P_{\mathbf{K}} = \{\varphi \in C_{\mathbf{r}}^{\alpha} : -\mathbf{K} \le \varphi \le \mathbf{K}, \rho(P(\mathbf{K})) < 1, [\varphi]_{2,\mathbf{r}}^{+} \in W_\rho(\mathbf{P}(\mathbf{K}))\},$$

with $[\varphi]_{2,\mathbf{r}}^{+} \equiv (\|\varphi_1\|_{2,r_1},\cdots,\|\varphi_m\|_{2,r_m})$, then for $\varphi \in W^e(0)$, there is positive constant λ such that the solutions $\mathbf{u}$ of the problem (1.1) satisfy

$$\|u_i(t)\|_2 \le \|\varphi_i\|_{2,r_i} e^{-\lambda t}, \ i = 1,\cdots,m, \quad t \ge 0. \tag{3.7}$$

Proof From $\rho(P(\mathbf{K})) < 1$, there exists sufficient small positive constant λ with $\lambda < \min_{1\le i\le m}\{\omega_i\}$ such that

$$\rho(\mathbf{P}(\mathbf{K}))\frac{\omega_i}{\omega_i - \lambda} < 1 \quad \text{for } i = 1,\cdots,m. \tag{3.8}$$

Multiplying (3.2) by $e^{\lambda t}$, it follows that, for $i = 1,\cdots,m$,

$$\begin{aligned}&\|u_i(t)\|_2 e^{\lambda t} \le e^{-(\omega_i-\lambda)t}\|\varphi_i(0,\cdot)\|_2 + \\ &\int_0^t e^{-(\omega_i-\lambda)(t-s)} e^{\lambda s}\{\frac{1}{|\Omega|}\int_\Omega [\sum_{j=1}^{m} p_{ij}([\mathbf{u}_s]_{\infty,\mathbf{r}}^{+})|u_{j,s}|_{r_j}]^2 \mathrm{d}x\}^{\frac{1}{2}}\mathrm{d}s, \ t \ge 0.\end{aligned} \tag{3.9}$$

We first show that, for $\varphi \in W^e(\mathbf{0})$, that is, there is a nonnegative row vector $\mathbf{K}$ such that $\mathbf{P}(\mathbf{K})\mathbf{K}^T \leq \mathbf{K}^T$, $-\mathbf{K} \leq \varphi \leq \mathbf{K}$, $[\varphi]^+_{2,\mathbf{r}} \in W_\rho(\mathbf{P}(\mathbf{K}))$ and $\rho(\mathbf{P}(\mathbf{K})) < 1$, it holds that

Case 1

$$\|u_i(t)\|_2 = 0, \quad i \notin \Gamma, \;\; t \geq 0; \tag{3.10}$$

Case 2

$$\|u_i(t)\|_2 e^{\lambda t} < d\|\varphi_i\|_{2,r_i}, \quad i \in \Gamma, \;\; t \geq 0, \tag{3.11}$$

where $d \in (1, \min_{i\in\Gamma}\{\frac{K_i}{\|\varphi_i\|_{2,r_i}}\})$ and $\Gamma = \{i : \|\varphi_i\|_{2,r_i} \neq 0, i = 1, \cdots, m\}$.

Case 1 Noticing that $[\varphi]^+_{2,\mathbf{r}} \in W_\rho(\mathbf{P}(\mathbf{K}))$ means

$$\sum_{j=1}^{m} \frac{p_{ij}(\mathbf{K})}{\omega_i}\|\varphi_j\|_{2,r_j} = \rho(\mathbf{P}(\mathbf{K}))\|\varphi_i\|_{2,r_i},$$

we have

$$p_{ij}(\mathbf{K}) = 0, \quad i \notin \Gamma, \; j \in \Gamma.$$

By Lemma 2.1, $\mathbf{u} \in < -\mathbf{K}, \mathbf{K} >$ when $-\mathbf{K} \leq \varphi \leq \mathbf{K}$. From (3.2), it follows that

$$\sum_{i\notin\Gamma} \|u_i(t)\|_2 \leq \sum_{i\notin\Gamma} \int_0^t e^{-\omega_i(t-s)} \sum_{j\notin\Gamma} p_{ij}(\mathbf{K})\|u_{j,s}\|_{2,r_j} \mathrm{d}s, \quad t \geq 0.$$

By Bellman inequality, we have

$$\|u_i(t)\| = 0, \quad i \notin \Gamma, \;\; t \geq t_0.$$

Case 2 If (3.11) is not true, then there must be some $l \in \Gamma$ and $t_2 > 0$ such that

$$\begin{aligned} &\|u_l(t_2)\|_2 e^{\lambda t_2} = d\|\varphi_l\|_{2,r_l}, \quad \|u_l(t)\|_2 e^{\lambda t} < d\|\varphi_l\|_{2,r_l} \;\text{ for }\; t < t_2, \\ &\|u_i(t)\|_2 e^{\lambda t} \leq d\|\varphi_i\|_{2,r_i} \qquad \text{for }\; t \leq t_2. \end{aligned} \tag{3.12}$$

Thus, it follows that

$$\begin{aligned} d\|\varphi_l\|_{2,r_l} &= \|u_l(t_2)\|_2 e^{\lambda t_2} \\ &\leq e^{-(\omega_l-\lambda)t_2}\|\varphi_l\|_{2,r_l} + \int_0^{t_2} e^{-(\omega_l-\lambda)(t_2-s)} \sum_{j=1}^{m} p_{lj}(\mathbf{K}) d\|\varphi_j\|_{2,r_j} \mathrm{d}s \\ &\leq e^{-(\omega_l-\lambda)t_2}\|\varphi_l\|_{2,r_l} + (1 - e^{-(\omega_l-\lambda)t_2}) d\sum_{j=1}^{m} \frac{p_{lj}(\mathbf{K})}{\omega_l - \lambda}\|\varphi_j\|_{2,r_j} \\ &\leq e^{-(\omega_l-\lambda)t_2}\|\varphi_l\|_{2,r_l} + (1 - e^{-(\omega_l-\lambda)t_2}) \frac{d\omega_l}{\omega_l - \lambda}\rho(\mathbf{P}(\mathbf{K}))\|\varphi_l\|_{2,r_l} \\ &< d\|\varphi_l\|_{2,r_l}, \end{aligned}$$

which is a contradiction, and so (3.1) holds. Letting $d \to 1$, we get

$$\|u_i(t)\|_2 \leq \|\varphi_i\|_{2,r_i} e^{-\lambda t}, \quad (i = 1, \cdots, m,) \quad t \geq 0. \qquad \square$$

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非线性抛物型偏泛函微分方程的渐近行为

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摘 要: 本文研究一类非线性抛物型偏泛函微分方程的渐近行为. 采用上下解方法，建立了其解的有界性和稳定性. 通过半群理论、非负矩阵性质和不等式技巧，得到估计这类方程平衡态渐近稳定域的方法.

关键词: 稳定域；有界性；上、下解.