The applications of nonlinear Kalman filters in passive tracking with bearing-only measurements

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Abstract: Target Motion Analysis (TMA) is one of the most popular techniques for estimation of time-varying state of underwater targets. Two types of nonlinear filters, Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF), are applied to the monostatic and bistatic bearing-only tracking. The tracking performance of the two filters is demonstrated via Monte Carlo simulations. The results show that the UKF outperforms in most cases in the underwater passive tracking scenarios, especially if the target makes maneuvers frequently. Compared with EKF, UKF shows better numerical stability and higher estimation accuracy with little cost of computational complexity.

Key words: target motion analysis; passive tracking; unscented Kalman filter

1 INTRODUCTION

In underwater passive tracking, the time-varying state of the targets such as location and velocity can be measured and estimated by several sonars mounted on one certain platform. For example, the hull-mounted array (HMA) sonar and the towed linear array (TLA) sonar are two of the standard devices for modern surface ships. In practice, it is not usually for the two types of sonars to work simultaneously. Hence, it is a much common tactical decision to detect and track the long-range targets by the monostatic sonar, TLA sonar for example. It is well known that the TLA sonar cannot provide the target's range information. One of the effective ways is to use the estimation techniques, such as Kalman Filtering.

In the bearing-only tracking (BOT), the information of target's bearing can only be used. In order to make the state equation of the system solvable, i.e. the system is observable, the following conditions must be satisfied:

(1) In monostatic passive tracking, it is necessary for the ownship to make maneuvers [1]. It can be proved that when the derivative of the ownship's trajectory is one order higher than that of the target, the system is observable[2].

(2) In bistatic passive tracking, the system is observable when the trajectory of the target does not superpose with the joint line between the two observers[2].

If the observability of the system is achieved, the state of the target can be solved by the estimators. Under the framework of Bayesian estimation theory, the full posterior probability density of the state given by all the observations constitutes a complete solution to the state estimation problem[3], so we can calculate out any optimal estimates of the state by recursively computing a marginal of the posterior filtering density. For a linear Gaussian tracking problem, the optimal Bayesian solution can be achieved by Kalman Filter (KF). But unfortunately, most of the tracking problems in practice are very complex, usually nonlinear, non-Gaussian, and non-stationary. This difference evokes model-mismatching and makes KF fail to provide satisfying solutions. Therefore, some other techniques have been adopted to handle this problem, among which the extended Kalman Filter (EKF) is the most famous one[4,5]. However, EKF approximates the nonlinearity

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by linearization, so that it is only effective in solving weak nonlinear problems. In the scenarios with strong nonlinearity, EKF-based algorithms usually give estimation with low accuracy, and the filter is much more instable and tends to be divergent[6].

In order to overcome the drawbacks of EKF, the Unscented Transform (UT)[7] has been adopted recently in KF to produce a type of new filter named Unscented Kalman Filter (UKF). UKF does not approximate the nonlinear functions by linearization as the EKF does. Instead, it approximates the posterior probability by a Gaussian density. The posterior probability can be represented by a set of deterministically samples. As a result, the statistics with the first two orders of the Gaussian density can be calculated by these samples, and the nonlinearity embedded in the state transition and mea-surements can be approximated up to the second order by UKF without any explicit information[6].

In this paper, UKF is used to handle the classic underwater BOT problem, and the results are compared with those of EKF.

2 TRACKING MODELS BASED ON BEARING MEASUREMENTS

2.1 Mathematical models

For a tracking problem, the state vector $\boldsymbol{X}_k \in \mathrm{R}^{n_x}$ is assumed to evolve according to the following discrete-time stochastic process

$$\boldsymbol{X}_k = f(\boldsymbol{X}_{k-1}, \boldsymbol{v}_k) \tag{1}$$

where $f: \mathrm{R}^{n_x} \times \mathrm{R}^{n_v} \to \mathrm{R}^{n_x}$ is a known, possibly nonlinear transition function of the system; $\boldsymbol{v}_k \in \mathrm{R}^{n_v}$ refers to the independent and identically distributed sampling sequence of the process noise with known probability density function. The state vector $\boldsymbol{X}_k$ is assumed to be independent from the process noise. Therefore, it is a first order Markovian process.

The measurements of $\boldsymbol{y}_k \in \mathrm{R}^{n_y}$ related to the target state are given by the measurement equation as follows:

$$\boldsymbol{y}_k = \boldsymbol{h}(\mathbf{X}_k, \boldsymbol{w}_k) \tag{2}$$

where $\boldsymbol{h}: \mathrm{R}^{n_x} \times \mathrm{R}^{n_w n} \to \mathrm{R}^{n_y}$ is the measurement function; $\boldsymbol{n}_k \in \mathrm{R}^{n_w}$ is the sample sequence of the measurement noise process which is independent from the states and the process noise, and can also be modeled as mutually independent and identically distributed with known probability density function.

Hence, the explicit forms of $\boldsymbol{f}$ and $\boldsymbol{h}$ are known, and the initial density function $p(\boldsymbol{X}_0|\boldsymbol{y}_0) \equiv p(\boldsymbol{X}_0)$ is also available. The states correspond to a Markov process and the measurements are conditionally independent of the given states.

2.2 General formulation

For two-dimensional (2D) underwater BOT problems, the measurements of the target are the bearings observed by spatially distributed sonars. Following Equation (1), the target dynamics can be described as

$$\boldsymbol{X}_k = \mathbf{A}\boldsymbol{X}_{k-1} + \mathbf{G}\boldsymbol{v}_k \tag{3}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 1 & \triangle t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & \triangle t \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 0.5\triangle t^2 & 0 \\ \triangle t & 0 \\ 0 & 0.5\triangle t^2 \\ 0 & \triangle t \end{bmatrix},$$

and

$$\boldsymbol{X}_k = \begin{pmatrix} x_k \\ \dot{x}_k \\ y_k \\ \dot{y}_k \end{pmatrix}^T \qquad \boldsymbol{v}_k = \begin{pmatrix} v_k^x \\ v_k^y \end{pmatrix},$$

where $\triangle t$ is the sampling interval; (x_k, y_k) is the position of target in Cartesian coordinates at instant k; $\dot{x}_k$ and $\dot{y}_k$ denote the components of velocity at instant k, respectively; $\boldsymbol{v}_k$ is a 2 by 1 i.i.d (independent identical distribution) process noise vector whose mean is zero and covariance is $\boldsymbol{Q}_k$.

The observers make noisy measurements of the target angle as follows

$$q_{k,i} = \tan^{-1}\left(\frac{x_k - x_{i,k}}{y_k - y_{i,k}}\right) + w_{k,i}, \quad i = 1, \cdots, N \tag{4}$$

where $\theta_{k,i}$ denotes the bearing of target obtained by the ith observer (with a number of N) at instant k; $(x_{i,k}, y_{i,k})$ is the position of the ith observer at instant k; $w_{k,i}$ is an N by 1 i.i.d measurement noise vector whose mean is zero and covariance is $\boldsymbol{R}_k$.

3 NONLINEAR FILTERING

In order to estimate the state vector as shown in Equation (1), the nonlinear filtering techniques should be adopted to handle the nonlinearity confronted in the bearing measurements described by Equation (3). The discussion of tracking methods will focus on the following two filters.

3.1 EKF

The EKF handles the nonlinearity by linearizing the nonlinear function to the first order, and then converting the nonlinear estimation problem into a linear one so that it can be solved via the standard KF. In BOT, the nonlinearity only exists in the measurement equation, which can be linearized as

$$\boldsymbol{\theta}_k=\boldsymbol{H}_k\boldsymbol{X}_k+\boldsymbol{w}_k \tag{5}$$

where $\boldsymbol{\theta}_k=[q_{1,k},\cdots,q_{N,k}]^T$, $\boldsymbol{w}_k=[w_{1,k},\cdots,w_{N,k}]^T$, $\boldsymbol{X}_k=[x_k,x_k,y_k,y_k]^T$, and the linearisation of measurement matrix $\boldsymbol{H}_k$ is given by

$$\boldsymbol{H}_k=\begin{bmatrix} \dfrac{\triangle y_{1,k|k-1}}{r^2_{1,k|k-1}}, & 0, & -\dfrac{\triangle x_{1,k|k-1}}{r^2_{1,k|k-1}}, & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\triangle y_{N,k|k-1}}{r^2_{N,k|k-1}}, & 0, & -\dfrac{\triangle x_{N,k|k-1}}{r^2_{N,k|k-1}}, & 0 \end{bmatrix} \tag{6}$$

where

$$\triangle x_{i,k|k-1}=x_{k|k-1}-x_{i,k};\quad \triangle y_{k|k-1}=y_{k|k-1}-y_{i,k}$$

$$r_{i,k|k-1}=\sqrt{(\triangle x_{i,k|k-1})^2+(\triangle y_{i,k|k-1})^2}$$

$(x_{k|k-1},y_{k|k-1})$ is the predicted position of the target at instant k, and $r_{i,k|k-1}$ is a relative distance associated with the ith observer. The posi-tions of the observers during the observation are assumed to be known.

The solution step of EKF formulated by Equation (3) and (5) resembles the standard Kalman Filter.

3.2 UKF

Unlike EKF, UKF captures the statistics by using the unscented transform to estimate the mean and covariance of a stochastic process. It abides by the principle that it is easier to approximate a probability distribution than an arbitrary nonlinear function[6]. The most computation-costly operation in UKF is the calculation of the square root of the covariance matrix of the state vector at each instant in order to form the square set.

Let $\boldsymbol{P}_k$, $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$ denote the covariance of $\boldsymbol{X}_k$, $\boldsymbol{v}_k$, $\boldsymbol{w}_k$ and with known initial values $\boldsymbol{P}_0$, $\boldsymbol{Q}_0$ and $\boldsymbol{R}_0$, respectively. And the estimation of the initial state $\boldsymbol{X}_0$ is also assumed to be available. Then, UKF algorithm can be implemented recursively in the following four steps.

3.2.1 Augmentation

At instant k, the augmented state vector $\boldsymbol{\chi}_k$ can be defined as

$$\boldsymbol{\chi}_k=[\boldsymbol{X}_k\ \ \boldsymbol{v}_k\ \ \boldsymbol{w}_k]^T \tag{7}$$

where $\boldsymbol{\chi}_k\in \mathrm{R}^n$, $n=n_x+n_v+n_w$.

Following the discussion presented in section 2.1, along with Equation (7), the covariance of $\boldsymbol{\chi}_k$ can be written as

$$\boldsymbol{\Phi}_k=\mathrm{diag}\{\boldsymbol{P}_k\ \ \boldsymbol{Q}_k\ \ \boldsymbol{R}_k\} \tag{8}$$

where $\boldsymbol{\Phi}_k$ denotes the covariance at instant k, and diag$\{\cdot\}$ denotes the block diagonal matrix.

The initial estimation of the augmented state vector and its covariance are given by

$$\hat{\boldsymbol{\chi}}_0=E[\boldsymbol{\chi}_0]=[\hat{\boldsymbol{X}}_0\ \ \boldsymbol{0}\ \ \boldsymbol{0}]^T$$
$$\boldsymbol{\Phi}_0=\mathrm{diag}\{\boldsymbol{P}_0,\boldsymbol{Q}_0,\boldsymbol{R}_0\} \tag{9}$$

where $\hat{\boldsymbol{X}}_0$ is the estimation of the mean of $\boldsymbol{X}_0$.

3.2.2 Calculation of the Sigma points and their weights

The $(2n+1)$ sigma points $\boldsymbol{X}_{i,k}$ and their weights $\boldsymbol{W}_i$ at instant k can be calculated by

$$\boldsymbol{X}_{0,k}=\boldsymbol{\chi}_k$$
$$W_0=\frac{\kappa}{n+\kappa}\ (i=0)$$
$$\boldsymbol{X}_{i,k}=\hat{\boldsymbol{\chi}}_k+(\sqrt{(n+\kappa)\boldsymbol{\Phi}_k})$$
$$W_i=\frac{1}{2(n+\kappa)}\qquad (i=1,\cdots,n)$$
$$\boldsymbol{X}_{i,k}=\hat{\boldsymbol{\chi}}_k-(\sqrt{(n+\kappa)\boldsymbol{\Phi}_k})$$
$$W_i=\frac{1}{2(n+\kappa)}\qquad (i=n+1,\cdots,2n) \tag{10}$$

where κ is a scaling parameter which controls the distance of the sigma points from the mean, and $(\sqrt{(n+\kappa)\boldsymbol{\Phi}_k})_i$ is the ith column of square root of $(n+\kappa)\boldsymbol{\Phi}_k$.

3.2.3 Prediction

The sigma points propagate through the nonlinear function

$$\begin{cases}\boldsymbol{X}_{i,k}=f(\boldsymbol{X}_{i,k})\\ \boldsymbol{z}_{i,k}=h(\boldsymbol{X}_{i,k})\end{cases},\qquad i=0,\cdots,2n \tag{11}$$

So, we can predict the mean and covariance as follows

$$\hat{\boldsymbol{X}}_{k+1|k}=\sum_{i=0}^{2n}W_i\cdot f(\boldsymbol{X}_{i,k})$$
$$\hat{\boldsymbol{z}}_{k+1|k}=\sum_{i=0}^{2n}W_i\cdot h(\boldsymbol{X}_{i,k})$$
$$\boldsymbol{P}_{zz}=\boldsymbol{R}_{k+1}+\sum_{i=0}^{2n}W_i(h(\boldsymbol{X}_{i,k})-\hat{\boldsymbol{z}}_{k+1|k})(h(\boldsymbol{X}_{i,k})-\hat{\boldsymbol{z}}_{k+1|k})^T$$
$$\boldsymbol{P}_{xz}=\sum_{i=0}^{2n}W_i(f(\boldsymbol{X}_{i,k})-\hat{\boldsymbol{X}}_{k+1|k})(h(\boldsymbol{X}_{i,k})-\hat{\boldsymbol{z}}_{k+1|k})^T \tag{12}$$

3.2.4 Update

Following equation (12), the filter gain can be calculated by

$$K_{k+1}=\boldsymbol{P}_{xz}\boldsymbol{P}_{zz}^{-1} \tag{13}$$

Then, the update procedure can be expressed by

$$\hat{\boldsymbol{X}}_{k+1|k+1}=\hat{\boldsymbol{X}}_{k+1|k}+K_{k+1}(\boldsymbol{z}_{k+1}+\hat{\boldsymbol{z}}_{k+1|k}) \tag{14}$$

$$\boldsymbol{P}_{k+1|k+1}=\boldsymbol{P}_{k+1|k}+K_{k+1}\boldsymbol{P}_{zz}\boldsymbol{K}_{k+1}^{T} \tag{15}$$

Note that UKF requires the computation of a matrix square root in (10), which can be done by using Cholesky factorisation.

4 NUMERICAL SIMULATIONS

In this section, EKF and UKF will be applied to BOT problems in monostatic and bistatic scenarios, respectively. The simulation conditions are given as follows:

The process noise $\boldsymbol{v}_k$ in Equation (3) and measurement noise $\boldsymbol{w}_k$ in Equation (4) are assumed to be additive Gaussian white noise with the constant covariance, respectively.

$$\boldsymbol{Q}=q_a\boldsymbol{I}_2$$
$$\boldsymbol{R}=q_\theta\boldsymbol{I}_M \tag{16}$$

where $\sqrt{q_a}=0.001$ km/s^2, $\sqrt{q_\theta}=0.01745$ rad. $\boldsymbol{I}_2$ and $\boldsymbol{I}_M$ are two identity matrix with the dimension of 2 and M (M=1 for monostatic, M=2 for bistatic).

The trajectories of the target (submarine) and ownship are shown in Fig.1 where the ownship makes maneuver to ensure the observability in monostatic scenario. In bistatic situation, the trajectory of ownship does not change for convenience to make comparisons. In both scenarios given above, the target makes maneuver with zigzag-like line to escape. The total observation period lasts 25 minutes during which the bearing of the target is measured every 30s (with the total measurements of 50). The initial state is estimated with respect to the realistic sonar device in an empirical way.

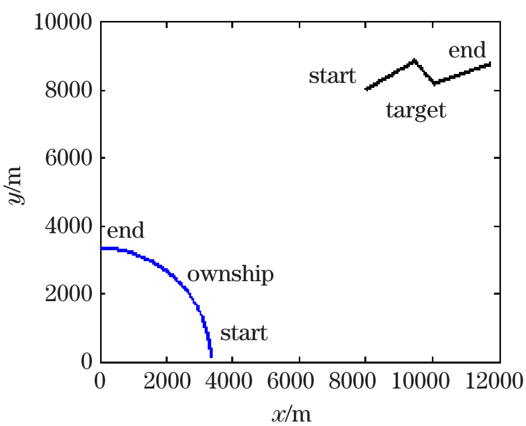

Fig.1　The trajectories of the observer and target

The performance of these two estimators is studied via Monte-Carlo simulations. The results of target state estimation are shown in Fig.2 and 3. The bearing of target is only observed by TLA sonar in the monostatic scenario, and by the HMA sonar and TLA sonar simultaneously in the bistatic scenario. As shown in Fig.2, the estimation error of both filters increases slowly with time and rises abruptly when the target maneuvers. The trajectory of the target can be estimated with acceptable error for underwater weapon guidance (usually several hundred meters for target positioning). UKF is more accurate than EKF, and shows good robustness for convergence.

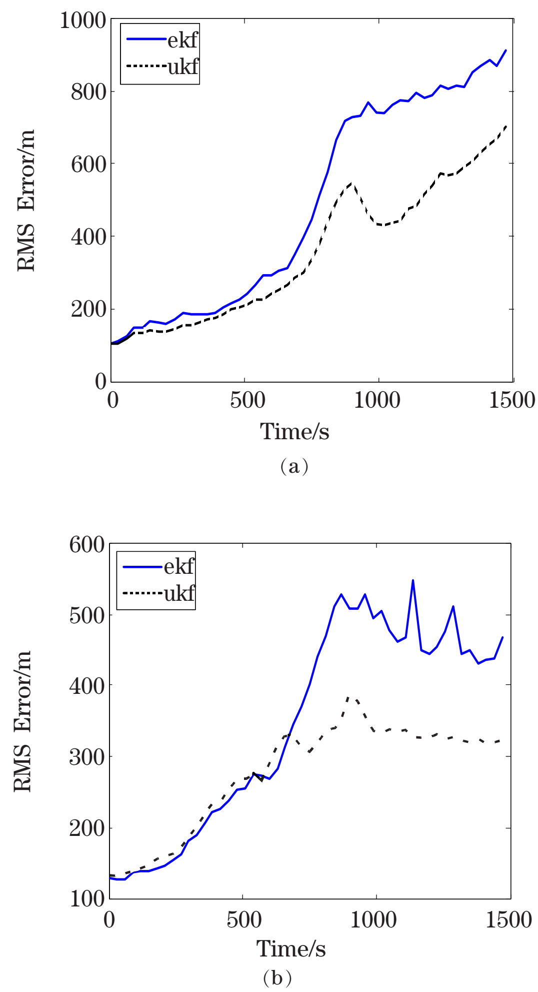

Fig.2　Square root of the estimated position of the target in the monostatic scenario: (a) position x; (b) position y. The bearings are measured every 30 seconds.

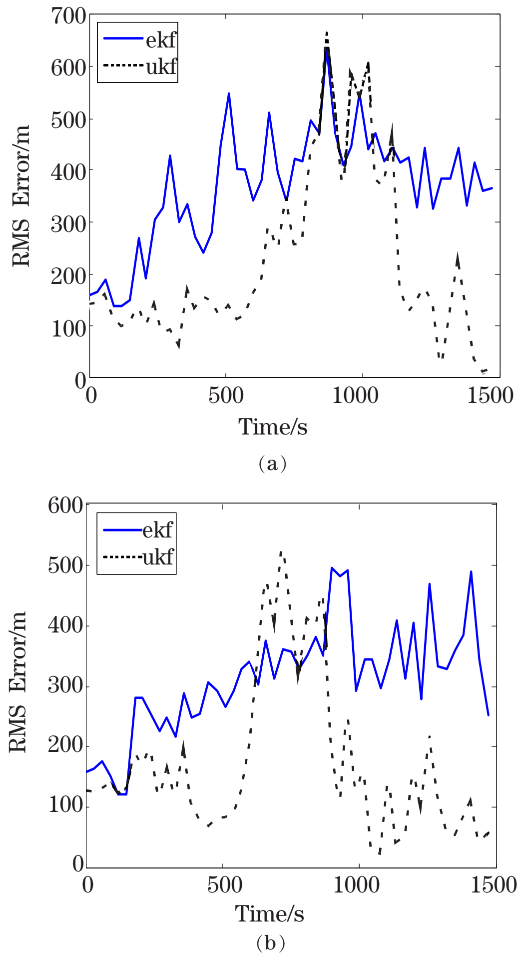

Fig.3 Square root of the estimated positions of the target in the bistatic scenario: (a) position x; (b) position y.

The results of bistatic estimation are shown in Fig.3. Compared with those of Fig.2, the estimation is better for increased observer numbers. Furthermore, the curves in Fig.3 are much rougher than those in Fig.2 due to the increasing number of divergence in solving UKF and EKF equations.

In both two scenarios, UKF acquires better accuracy of state estimation, and is more robust to avoid numerical divergence, with the cost of small increase in computational complexity (about twice of EKF).

5 CONCLUSIONS

In this paper, two types of nonlinear filters have been applied to the BOT problems in the monostatic and bistatic scenarios. The performance of these two estimators is evaluated by the Monte-Carlo simulations. The results show that algorithm of UKF outperforms the EKF in terms of accuracy and stability. With slightly increasing in computational cost, the UKF deserves to be a promising and valuable tool in the underwater TMA issues.

References

[1] Nardone S C, Aidala V J. Observability Criteria for Bearing-Only Target Motion Analysis [J]. IEEE Trans. on Aerospace and Electronic Systems, 1981, **17**(2): 162-166.

[2] Jauffret C, Pillon D. Obserability in Passive Target Motion Analysis [J]. IEEE Trans. on Aerospace and Electronic Systems, 1996, **32**(4): 1290-1300.

[3] Blackman S, Popoli R. Design and Anlysis of Modern Tracking Systems[M]. Norwood, MA: Artech House, 1999.

[4] Song T A, Speyer J L. A stochastic analysis of the modified gain extended Kalman filter with application to estimation with bearings-only measurements[J]. IEEE Trans. Automat. Contr., 1985, **30**: 940-949.

[5] 石章松, 王树宗. 直角坐标系下的水下被动目标跟踪自适应卡尔曼滤波算法[J]. 声学技术, 2004, **23**(3): 173-177.
SHI Zhangsong, WANG Shuzong. An adaptive Kalman filter used in the underwater passive target tracking in a Cartesian coordinate system[J]. Technical Acoustics, 2004, **23**(3): 173-177.

[6] Ristic B, Arulampalam M S. Tracking a manoeuvring target using angle-only measurements: algorithms and performance[J]. Signal Processing, 2003, **83**: 1223-1238.

[7] Julier S J, Uhlmann J K. Unscented filtering and non-linear estimation[J]. IEEE Proceedings, 2004, **92**(3): 401-422.

非线性 Kalman 滤波器在纯方位被动跟踪中的应用

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摘要: 目标运动分析(简称 TMA)是用于估计水下目标时变状态最主要的技术之一。应用了两种非线性滤波器——EKF和 UKF 来估计单/双基地情况下的目标运动状态,并由蒙特-卡洛仿真给出其跟踪性能。数值结果表明:在大部分情况下,特别是当目标存在机动时,UKF 在估计精度和数值稳定性上都要好于 EKF,其代价仅是少量地增加了的计算复杂度。

关键词: TMA;被动跟踪;UKF

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