

# Radiation force calculation of focused ultrasound and its experiment in high intensity focused ultrasound

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**Abstract:** This paper studies the method of calculating the acoustic power of the focused ultrasonic beam based on the radiation force. The general formula of estimating the radiation forces acting on the partially reflecting targets in the focused ultrasonic field of a spherical zone transducer were derived from the ray acoustics. The effects of radiation forces acting on the totally reflecting and absorbing targets were discussed. A radiation force balance (RFB) for measuring the acoustic power of high intensity focused ultrasound was also established. The difference between the measured value by calorimetry method and that derived using the studied method was found to be less than 3%.

**Key words:** focused ultrasound; radiation force; calculation; acoustic power; measurement

## 聚焦超声的辐射力计算与高强度聚焦 超声功率测量实验

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**摘要:** 研究聚焦超声场的辐射力计算。应用几何声学方法, 推导了聚焦超声作用于测试靶上的辐射力通用公式, 讨论了全反射靶和全吸收靶上的辐射力。最后给出了应用辐射力法测量高强度聚焦超声装置的声功率的实例, 其结果与良热法测得的声功率接近, 偏差不大于 3%。

**关键词:** 聚焦超声; 辐射力; 计算; 测量

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## 1 INTRODUCTION

The measurement of ultrasound power in fluid is through the observation of radiation forces<sup>[1,2]</sup>. This method is widely used and published in the IEC

standards<sup>[3,4]</sup>. This method is mainly focused at the measurement of plan wave ultrasound power. The formula for calculation of radiation force acting on a totally absorbing target in the focused ultrasound field derived by K. Beissner is based on a series of assumptions (i.e., ray acoustics, non-diffraction high frequency limitation of time, far-field directivity is re-ctangle function). The radiation force which acts on an absorbing target in the focused ultrasonic

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field is calculated using Rayleigh integral. When  $ka$  is approaching  $30^\circ$  and the half-aperture angle is  $30^\circ$ ; the deviation is less than 0.8% ( $k$  is circular wave number, and  $a$  is the half-aperture of the focused transducer). This means that the theoretical assumption is reasonable. Based on the acoustic geometry method, a general formula of calculating the radiation force acting on the target within the focused ultrasound field was established and it was also validated with several measured values in the subsequent experiments.

## 2 THE PRINCIPLE

The radiation force of a spherical shaped zone induced from the focusing ultrasonic transducer was studied. Its inner and outer apertures are represented by  $2b$  and  $2a$  respectively and  $ka > kb \gg 10$ . The transducer produces ideal focused sound field and satisfies the assumption of ray acoustics: the sound intensity on the transducer's surface is a constant,  $I_0$ ; the focus spot diameter is zero; the ultrasound energy has no loss in propagation; the acoustic energy flow is identical in the same solid angle passing the focus. The radius of curvature of transducer or focal length is  $R$ . The halfaperture angles of two edges of the spherical zone are  $\alpha_1 = \arcsin(b/R)$  and  $\alpha_2 = \arcsin(a/R)$  respectively. The configuration is illustrated in Fig.1.

A surface area element,  $ds$  is perpendicular to the ultrasound ray at point  $Q$  where the acoustic intensity is  $I$  in the sound field. The solid angle constructed by focus  $O$  and  $ds$  extends from the focus to the surface of the sound source and forms a projected surface area element,  $ds$  on the radiation face of the transducer. Thus,

$$I \times ds = I_0 \times ds \tag{1}$$

According to Langiven's radiation pressure principle, the impulse flow energy density of a plane wave, i.e. radiation pressure  $Pr$ , can be expressed as:

$$Pr = I/c \tag{2}$$

Where,  $I$  is the acoustic intensity of a plane wave,

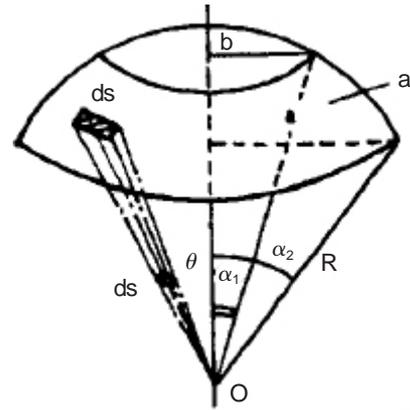


Fig.1 Focused ultrasound field of spherical shaped transducer

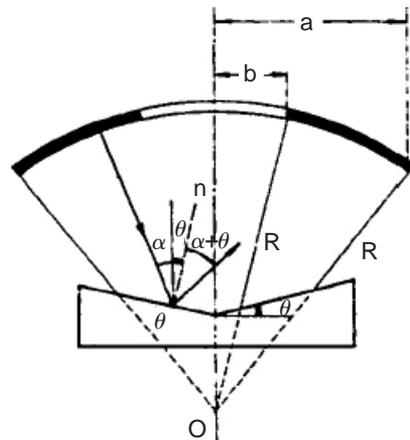


Fig.2 The radiation force produced by focused ultrasound field of spherical zone-shaped transducer acting on a concave conical target

in  $W/m^2$ .  $c$  is the sound velocity in medium (water), in  $m/s$ .

A concave conical surface target is located at a position between the transducer and the focus in the above sound field which should be large enough to intersect all sound energy. The angle between the symmetric axis and normal of the surface of the target is  $\theta$ . The symmetrical axis of the target is aligned to symmetrical axis of the surface of transducer. The arrangement is illustrated in Fig.2. Assuming a sound ray with an acoustic intensity is  $I$ , which injects to a point  $Q$  on the target, the sound ray can be looked as a very thin sound beam of plane wave. The angle between the sound ray and the symmetrical axis (or the beam axis) is  $\alpha$ . The incidence angle of the sound ray is therefore equal to  $(\alpha + \theta)$ .

At point Q, the axial component of the incidence sound impulse flow energy density is,

$$I \times \cos\alpha/c$$

and the axial component of the reflecting sound impulse flow energy density is

$$r^2 \cos(\alpha+2\theta)/c,$$

where,  $r$  is the pressure amplitude reflection coefficient on the interface between water and the target.

According to the impulse conservation law, the axial component of impulse flow acting at point Q on reflecting target for a plane wave can be expressed as:

$$Pr=(1/c) [\cos\alpha+r^2\cos(\alpha+2\theta)] \quad (3)$$

The whole focused sound beam is made up of numerous thin sound beams. The cross-section of a thin sound beam at the incidence point Q is  $ds$  and each thin sound beam can be seen as a thin plane wave beam. The beam acting on the reflecting target can be treated as an axial component of the radiation force caused by the thin sound beam. It can be expressed as:

$$dF=(1/c) [\cos\alpha+r^2\cos(\alpha+2\theta)] ds \quad (4)$$

It is assumed that the surface area element  $ds$  is perpendicular to the incidence sound ray at point Q and the another surface area element  $ds$  on the radiation surface of the transducer is in the same focused solid angle. According to Equation(1), the axial component of the radiation force at point Q can be expressed as:

$$dF=(1/c) [\cos\alpha+r^2\cos(\alpha+2\theta)] ds \quad (5)$$

The axial radiation force acting on the whole reflecting target equals to the surface integral of

$$dF \text{ on } ds \text{ or on } ds, \text{ i.e. } F=\int_S dF=\int_S dF.$$

The perpendicular component of radiation force acting on the target sums up to be equal to zero because the surfaces of the transducer and target are symmetrical. In the spherical coordinate system, the sum of axial component of radiation force acting on the target, i.e. the total axial radiation force, can be calculated from the following equation:

$$F=2\pi R^2(1/c) \int_{\alpha_1}^{\alpha_2} [\cos\alpha+r^2\cos(\alpha+2\theta)] \sin\alpha d\alpha$$

Considering the acoustic power of the sound source,  $P=2\pi R^2(I_0/c) (\cos\alpha_1-\cos\alpha_2)$ , the result of integral obtained from above equation can be expressed as:

$$F=[(1+r^2\cos 2\theta) (\cos 2\alpha_1-\cos 2\alpha_2)+r^2\sin 2\theta (2\alpha_1-\sin 2\alpha_1-2\alpha_2+\sin 2\alpha_2)] P/[4c(\cos\alpha_1-\cos\alpha_2)]$$

or

$$P=4Fc(\cos\alpha_1-\cos\alpha_2)/[(1+r^2\cos 2\theta) (\cos 2\alpha_1-\cos 2\alpha_2)+r^2\sin 2\theta (2\alpha_1-\sin 2\alpha_1-2\alpha_2+\sin 2\alpha_2)] \quad (6)$$

Using a totally reflecting target ( $r=1$ ), the following equation can be obtained from Equation (6).

$$P=4Fc(\cos\alpha_1-\cos\alpha_2)/[(1+\cos 2\theta) (\cos 2\alpha_1-\cos 2\alpha_2)+(2\alpha_1-\sin 2\alpha_1-2\alpha_2+\sin 2\alpha_2)\sin 2\theta] \quad (7)$$

Using a spherical segment-shaped transducer ( $\alpha_1=0$ ) and a totally reflecting target ( $r=1$ ), the following equation can be obtained from Equation (6).

$$P=Fc(1-\cos\alpha_2)/[\sin^2\alpha_2\cos^2\theta+(\sin\alpha_2\cos\alpha_2-\alpha_2)\sin\theta\cos\theta] \quad (8)$$

Note: For convex conical reflecting target, the angle in above formulas should be a negative angle.

Using a baffled piston transducer ( $\alpha_1=\alpha_2=0$ ) and totally reflecting target ( $r=1$ ), the following equation can be obtained from Equation (7).

$$P=Fc/2\cos^2\theta \quad (9)$$

Using a spherical zone-shaped haped transducer and a totally absorbing target ( $r=0$ ), the following equation can be obtained from Equation (6).

$$P=2Fc/(\cos\alpha_1+\cos\alpha_2) \quad (10)$$

Using a spherical segment-shaped transducer ( $\alpha_1=0$ ) and a totally absorbing target ( $r=0$ ), the following equation can be obtained from Equation (10).

$$P=2Fc/(1+\cos\alpha_2) \quad (11)$$

This result is identical to that derived by K. Beissner<sup>[9]</sup>. Using a baffled piston transducer ( $\alpha_1=\alpha_2=0$ ) and totally absorbing target ( $r=0$ ), the following equation can be obtained from Equation (10).

$$P=Fc \quad (12)$$

### 3 VALIDATION EXPERIMENT

The measurements of ultrasonic power produced by the high intensity focused ultrasound system

were conducted in collaboration with the Department of Biomedical Engineering at Shanghai Jiaotong University and the Institute of Medical Ultrasound Engineering at Chongqing University of Medical Science. The ultrasound system is used for the experiment of abortion of animal at 1.6MHz. The sound field produced by the ultrasonic focusing transducer of the equipment is equivalent to that produced by a spherical zone-shaped ultrasonic focusing transducer. Its inner and outer half aperture angles are  $14.81^\circ$  and  $37.84^\circ$  respectively. The  $ka$  and  $kb$  values are 896 and 167 respectively.

Using a totally absorbing target to measure the acoustic power, we can obtain the following equation  $P=1.139Fc$  according to Equation (10). The result of measurement is 154.4W. The result is close to 159W which was obtained by the Institute of Medical Ultrasonic Engineering at Chongqing University of Medical Science and the Institute of Acoustics

at Nanjing University who use calorimetry method. The deviation is less than 3%. It shows that the formula derived above is available for used to estimate the sound power parameters of high frequency and high intensity focused ultrasound field. Using high frequency and large aperture ultrasonic focusing transducer, the results will be more accurate.

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#### 简讯

### 上海市声学学会生理声学专业委员会召开学术会

2006年11月23日下午,上海市声学学会生理声学专业委员会在上海市第六人民医院举行学术会议,会议由范静平、殷善开主持。有来自上海市的34名耳鼻咽喉科医师及听力学研究者参加,会上分别由仁济医院、海军医学研究所、长征医院和第六人民医院专家作生理声学专题报告,并进行了热烈讨论,对进一步促进上海地区生理声学的研究有重要意义。

长征医院 范静平