Vol. 36 (2016)
No. 3

数 学 杂 志
J. of Math. (PRC)

ASYMPTOTIC PROPERTIES FOR WAVELET ESTIMATOR OF REGRESSION FUNCTION BASED ON PA ERRORS

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Abstract: In this article, we study the asymptotic properties for wavelet estimator of regression function. By using the method of the probability inequalities, we obtain the r-moment convergence, consistency and asymptotic normality for the wavelet estimator of $g(\cdot)$, which generalize the corresponding results for mixing dependent random sequences.

Keywords: wavelet estimator; positively associated; consistency; asymptotic normality

2010 MR Subject Classification: 60F15

Document code: A **Article ID:** 0255-7797(2016)03-0533-10

1 Introduction

Consider the estimation of a standard nonparametric regression model involving an regression function $g(\cdot)$ which is defined on [0,1]

$$Y_i = g(t_i) + \varepsilon_i \ (1 \le i \le n), \tag{1.1}$$

where $\{t_i\}$ are non-random design points, denoted by $\{t_i\}$ and taken to be ordered $0 \le t_1 \le \cdots \le t_n \le 1$, $\{\varepsilon_i\}$ are random errors.

It is well known that regression function estimation is an important method in data analysis and has a wide range of applications in filtering and prediction in communications and control systems, pattern recognition and classification, and econometrics. So model (1.1) was studied extensively.

For model (1.1), the estimator of $g(\cdot)$ is defined as

$$g_n(t) = \sum_{i=1}^{n} Y_i \int_{A_i} E_m(t,s)ds, \tag{1.2}$$

* **Received date:** 2014-07-19 **Accepted date:** 2015-03-19

Foundation item: Supported by the National Natural Science Foundation of China (11461057); the Natural Science Foundation of Guangxi (2014GXNSFBA118011); the Science Foundation of Guangxi Education Department (ZD2014120)

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The wavelet kernel $E_m(t,s)$ can be defined as follows: $E_m(t,s) = 2^m E_0(2^m t, 2^m s) = 2^m \sum_{k\in Z} \phi(2^m t-k)\phi(2^m s-k)$, $\phi(\cdot)$ is a scaling function, where $A_i=[s_{i-1},s_i]$ is a partition of interval [0,1], $s_i=(1/2)(t_i+t_{i+1})$, and $t_i\in A_i, 1\le i\le n$.

We knew that wavelets was used widely in many engineering and technological fields, especially in picture handling by computers. In order to meet practical demands, since the 90s of the 20th century, some authors considered using wavelet methods in statistics.

It was well known that wavelet estimation methods was studied extensively, for instance, Antoniadis et al. (1994) introduced wavelet analogues of some familiar kernel and orthogonal series estimators, studied their finite sample and asymptotic properties, and discovered that is a fundamental instability in the asymptotic variance of wavelet estimators caused by the lack of translation invariance of wavelet transform; Sun and Chai (2004) on the α-mixing stationary process considered the same nonparametric regression model in this paper, the authors adopted wavelet method to estimate $g(\cdot)$ and studied it's consistency, strong consistency and convergence rate; Liang and Wang (2010) used the wavelet method to study semi-parametric regression model $y_i = x_i\beta + g(t_i) + V_i$, $(V_i = \sum_{j=-\infty}^{\infty} c_j e_{i-j})$, and obtained reasonable results; Hu et al. (2013), using the wavelet method, obtained some estimators of the parametric component, the nonparametric component and the variance function, investigated the asymptotic normality and weak consistence rates of these wavelet estimators. In Lu and Tao's (2012) a new wavelet-based algorithm was developed using log-linear relationship between the wavelet coefficient variance and the scaling parameter.

Definition 1.1 A finite family of random variables $\{Y_j, 1\le j\le n\}$ is said to be positively associated (PA). If for every pair of disjoint subsets A_1 and A_2 of $\{1,\cdots,n\}$, it holds that

$$\mathrm{Cov}\{g_1(Y_i, i\in A_1), g_2(Y_j, j\in A_2)\}\ge 0,$$

where g_1 and g_2 are nondecreasing coordinate wise for every variable and such that covariance exists. Infinite families of random variables are said to be PA, if any finite subset of them is a set of PA random variables.

The definition of PA random variables was introduced by Esary et a1. (1967), who studied it in detail. It is well known that PA random variables are widely encountered in applications, for example, in reliability theory, in mathematical physics and in percolation theory. For a recent review of this concept along with many probabilistic and statistical results, Yang and Li (2005) discussed the uniformly asymptotic normality of the nonparametric regression weighted estimator in positively associated samples and gave the rates of the uniformly asymptotic normality; Li et a1. (2008) studied uniformly asymptotic normality of wavelet estimator of regression function, the rates of uniformly asymptotic normality were shown as $O(n^{-1/6})$; Xing and Yang (2011) discussed strong convergence rate for positively associated random variables and gave the strong convergence rate; Li and Li (2013), using the properties of positively associated random variables, obtained the precise asymptotic for moving average processes, the results are some generalizations of previous results for moving average processes based on negatively associated random variables.

In this paper, we aim to discuss the asymptotic properties for wavelet estimator of a nonparametric fixed design regression function when errors are strictly stationary and PA random variables.

2 Assumptions and Main Results

In order to list some restrictions for φ and g, we give two definitions here.

Definition 2.1 Function φ is said to be τ -regular ($\varphi \in S_\tau, \tau \in N$) if for any $l \leq \gamma$ and any integer k, one has $|\frac{\partial^l_\varphi}{dx^l}| \leq C_k(1+|x|)^{-1}$, where C_k is a constant depending only on k.

Definition 2.2 A function space $H^\nu (\nu \in R)$ is said to be Sobolev space of order V, i.e., if $h \in H^\nu$ then $\displaystyle\int |\hat{h}(w)|^2(1+w^2)^\nu dw < \infty$, where $\hat{h}$ is the Fourier transform of h.

Some basic assumptions

(A1) $g(\cdot) \in H^\nu,\ \nu > 1/2$, and $g(\cdot)$ satisfy the Lipschitz condition of order 1;

(A2) $\varphi(\cdot) \in S_\tau$, and $\varphi(\cdot)$ satisfy the Lipschitz condition with order 1 and $|\hat{\varphi}(\varepsilon) - 1| = O(\varepsilon)$ as $\varepsilon \to \infty$, where φ is the Fourier transform of φ;

(A3) $\max\limits_{1\leq i\leq n} |s_i - s_{i-1}| = O(n^{-1})$;

(A4) (i) for each n, the joint distribution of $\{\varepsilon_i; i = 1, \cdots n\}$ is the same as that of $\{\xi_1; , \cdots \xi_n\}$, where $\{\xi_i; i = 1, \cdots n\}$ is PA time series with zero mean and finite second moment, $\sup\limits_{j\geq 1} E(\xi_j^2) < \infty$; (ii) $\sup\limits_{j\geq 1} E(\xi_j^{2+\delta}) < \infty$ for some $\delta > 0$;

(A5) $u(q) = \sup\limits_{i\in N} \sum\limits_{j:|j-i|\geq q} |\mathrm{Cov}(\varepsilon_i, \varepsilon_j)|$, with $u(1) < \infty$;

(A6) there exist positive integers $p := p(n)$ and $q := q(n)$ such that $p + q \leq n$ for sufficiently large n and as $n \to \infty$, (i) $qp^{-1} \to 0$, (ii) $pn^{-1} \to 0$.

Our main results are as follows.

Theorem 2.1 Let $\{\varepsilon_i; 1 \leq i \leq n\}$ be PA errors with mean zero and $\sup\limits_{1\leq i\leq n} E\varepsilon_i^2 = \sigma^2 < \infty$. Assume that assumptions (A1)–(A4) are satisfied. When $0 < r \leq 2$, then

$$E|g_n(t) - g(t)|^r = O(n^{-r}) + O(\eta_m^{-r}) + O(2^{2m}/n)^{r/2}.$$

Furthermore, let $2^{2m}/n \to 0$. When $0 < r \leq 2$, then

$$\lim_{n\to\infty} E|g_n(t) - g(t)|^r = 0.$$

Theorem 2.2 Let $\{\varepsilon_i; 1 \leq i \leq n\}$ be identically distributed PA errors, and $E|\varepsilon_i| = O(i^{-(1+2\rho)})$, $E\varepsilon_i^2 = \sigma^2 < \infty$. Assume that assumptions (A1)–(A3) hold, and $2^m = O(n^{1-\tau})$ for $1/2 < \tau < 1$, then

$$\sup_{0\leq t\leq 1} |g_n(t) - g(t)| \xrightarrow{p} 0.$$

Theorem 2.3 Let $\{\varepsilon_i; 1 \leq i \leq n\}$ be PA errors with mean zero, assume that assumptions (A1)–(A3) hold, and $2^m = O(n^{1-\tau})$ for $1/2 < \tau < 1$, then

$$g_n(t) \to g(t) \ \text{ a.s..}$$

Theorem 2.4 Let $g: A \to R$ is a bounded function defined on the compact subset A of R^d and (A1)–(A6) hold, we have

$$\sigma_n^{-1}(t)\{g_n(t)-g(t)\} \xrightarrow{d} N(0,1).$$

Remark 2.1 Condition (A1)–(A3) are general conditions of wavelet estimation, see [2–5]; Condition (A4)–(A5) are the same basic assumption as that used in literature (Yang (2005)), this shows that both the weighted function estimation and wavelet estimation can use the same assumption; proper selection of p, q for condition (A6) is easy to be satisfied, thus assumption is reasonable, which is the same as the literature (Yang (2005)) too, so we can see that the conditions in this paper are suitable and reasonable.

Remark 2.2 (I) Theorem 2.2 generalize and extend Theorem 2.1 of Li et al. (2008) from the weak consistency to uniformly weak consistency, it meas Theorem 2.2 will be satisfied under PA.

(II) Theorem 2.3 of Li et al. (2008) using general method discussed uniformly asymptotic normality of wavelet estimator of regression function, the rates of uniformly asymptotic normality were $O(n^{-1/6})$, but Theorem 2.4 use theorem condition of Lyapunov central limit to discuss uniformly asymptotic normality. Under the same condition, we get more ideal results. So Theorem 2.4 improves and extends the corresponding results in Theorem 2.3 of Li et al. (2008).

3 Some Lemmas

In order to prove our main results, we need the following lemmas.

Lemma 3.1 Under assumptions (A1)–(A3), we have

(I) $\sup\limits_{x,m}\int_0^1 |E_m(x,y)|dy < \infty$;

(II) $\int_0^1 |E_m(t,s)|g(s)ds = g(t) + O(\eta_m)$, where

$$\eta_m = \begin{cases} (1/2^m)^{\nu-1/2}, & 1/2 < \nu < 3/2, \\ \sqrt{m}/2^m, & \nu = 3/2, \\ 1/2^m, & \nu > 3/2; \end{cases}$$

(III) $|\int_{A_i} E_m(x,y)ds| = O(2^m/n), \quad i = 1, \cdots, n$;

(IV) $\sum\limits_{i=1}^n(\int_{A_i} E_m(x,y)ds)^2 = O(2^m/n)$.

Proof The proofs of (I) and (II) can see Antoniadis et al. (1994), and (III) and (IV) can be found in Lemma 2.1(3) of Sun and Chai (2004).

Lemma 3.2 Let assumptions (A1)–(A3) hold, and $\{\varepsilon_i; 1 \le i \le n\}$ be PA random variables with zero means, then

(I) $Eg_n(t) - g(t) = O(\eta_m) + O(n^{-1}), \quad \lim\limits_{n\to\infty} Eg_n(t) = g(t)$;

(II) $\sup\limits_{0\le t\le 1}|Eg_n(t)-g(t)|=O(\eta_m)+O(n^{-1}),\quad \lim\limits_{n\to\infty}\sup\limits_{0\le t\le 1}|Eg_n(t)-g(t)|=0.$

Proof Follows immediately from Lemma 3.1 and the method used for proving Lemma 3.1 of Sun and Chai (2004).

Lemma 3.3 (see Li (2003)) Let $\{Z_i, i\ge 1\}$ be random variables, if there exists a constant $\rho>0$, with $E|Z_i|=O(i^{-(1+2\rho)})$, then $\sum\limits_{i=1}^{\infty} Z_i$ a.s. convergence.

Lemma 3.4 (see Yang (2005)) Let $\{\xi_j, j\ge 1\}$ be steady PA random variables with zero means and $\sup\limits_{j\ge 1}E(\xi_j^2)<\infty$, when $r>2$ and $\delta>0$, $\sup\limits_{j\ge 1}E|\xi_j|^{r+\delta}<\infty$, $u(n)=O(n^{-(r-2)(r+\delta)/(2\delta)})$. Also $\{a_j, j\in N\}$ is a sequence of real numbers, $a:=\sup|a_j|<\infty$. Then

$$E\left|\sum_{j=1}^{n}a_j\xi_j\right|^r\le Ca^rn^{r/2}.$$

Lemma 3.5 (see Yang (2005)) Let $\{X_j : j\ge 1\}$ be a sequence of associated PA random variables, and let $\{a_j : j\ge 1\}$ be a real constant sequence, $1=m_0<m_1<\cdots<m_k=n$. Denote by $Y_l:=\sum\limits_{j=m_{l-1}+1}^{m_l}a_jX_j$ for $1\le l\le k$. Then

$$\left|E\exp\left(it\sum_{l=1}^{k}Y_l\right)-\prod_{l=1}^{k}E\exp(itY_l)\right|\le 4t^2\sum_{1\le s\le j\le n}|a_sa_j||\mathrm{Cov}(X_s,X_j)|.$$

4 Proofs of the Main Results

Proof of Theorem 2.1 By using the C_r -inequality and Jensen inequality for $0<r\le 2$, we have

$$\begin{aligned} E|g_n(t)-g(t)|^r &\le 2^{r-1}[|Eg_n(t)-g(t)|^r+E|g_n(t)-Eg(t)|^r] \\ &\le 2^{r-1}[|Eg_n(t)-g(t)|^r+C|\mathrm{Var}(g_n(t))|^{r/2}]. \end{aligned}\tag{4.1}$$

From Lemma 3.3 (I), we have

$$|Eg_n(t)-g(t)|^r=O(n^{-r})+O(\eta_m^{-r}).\tag{4.2}$$

Assume that assumptions (A1)–(A4) and Lemma 3.1 hold, we have

$$\begin{aligned} \mathrm{Var}(g_n(t)) &= \sigma_n^2\sum_{i=1}^{n}\left(\int_{A_i}E_m(t,s)ds\right)^2+O(2^{2m}/n)\sum_{1\le i<j\le n}\mathrm{Cov}(\varepsilon_{ni},\varepsilon_{nj}) \\ &= O(2^{2m}/n)+O(2^{2m}/n)\sum_{i=1}^{n-1}\sum_{j=i-1}^{n}\mathrm{Cov}(\varepsilon_{ni},\varepsilon_{nj}) \\ &= O(2^{2m}/n). \end{aligned}\tag{4.3}$$

Therefore, the conclusion follows from relations (4.1)–(4.3) and $2^{2m}/n\to 0$.

Proof of Theorem 2.2 We write

$$\sup_{0\le t\le 1}|g_n(t)-g(t)| \le \sup_{0\le t\le 1}|g_n(t)-Eg_n(t)| + \sup_{0\le t\le 1}|Eg_n(t)-g(t)| := I_{n3}+I_{n4}. \tag{4.4}$$

By Lemma 3.2 (II), we have

$$I_{n4}\to 0\ \ (n\to\infty). \tag{4.5}$$

Now, I_{n3} can be decomposed as

$$\begin{aligned} I_{n3} &= \sup_{0\le t\le 1}\left|\sum_{i=1}^n \varepsilon_i\int_{A_i}E_m(t,s)ds\right| = \sup_{0\le t\le 1}\left|\sum_{i=1}^n \varepsilon_i\int_{A_i}E_m(t,s)ds\sum_{j=1}^n I(s_{j-1}<t\le s_j)ds\right| \\ &\le \sup_{0\le t\le 1}\left|\sum_{i=1}^n \varepsilon_i\int_{A_i}\sum_{j=1}^n(E_m(t,s)-E_m(t_j,s))I(s_{j-1}<t\le s_j)ds\right| \\ &\quad+\sup_{0\le t\le 1}\left|\sum_{i=1}^n \varepsilon_i\int_{A_i}\sum_{j=1}^n E_m(t,s)dsI(s_{j-1}<t\le s_j)ds\right| := I_{n31}+I_{n32}. \end{aligned}\tag{4.6}$$

By condition of Theorem 2.2 and Lemma 3.3 we have

$$\begin{aligned} &I_{n31}\le \sup_{0\le t\le 1}\sum_{i=1}^n|\varepsilon_i|\int_{A_i}\sum_{j=1}^n|E_m(t,s)-E_m(t_j,s)|I(s_{j-1}<t\le s_j)ds \\ \le\ & C\sup_{0\le t\le 1}\left(\sum_{j=1}^n|\varepsilon_i|\int_{A_i}\sum_{j=1}^n(2^{2m}/n)I(s_{j-1}<t\le s_j)ds\right)\le\sum_{i=1}^n\frac{|\varepsilon_i|}{n^{4/3}}=O(n^{-4/3}), \quad (4.7) \\ &I_{n32}\le\sup_{0\le t\le 1}\left|\sum_{j=1}^n I(s_{j-1}<t\le s_j)\right|\left|\sum_{i=1}^n\varepsilon_i\int_{A_i}E_m(t_j,s)ds\right| \\ \le\ & \sup_{0\le t\le 1}\sum_{j=1}^n I(s_{j-1}<t\le s_j)\max_{1\le j\le n}\left|\sum_{i=1}^n\varepsilon_i\int_{A_i}E_m(t_j,s)ds\right| \\ \le\ & \max_{1\le j\le n}\left|\sum_{i=1}^n\varepsilon_i\int_{A_i}E_m(t_j,s)ds\right|. \end{aligned}$$

By the Markov inequality, Lemma 3.1 and Lemma 3.4, we have

$$\begin{aligned} P(|I_{n32}|\ge n^{-1/12}) &\le \sum_{j=1}^n P\left(\sum_{i=1}^n|\varepsilon_i\int_{A_i}E_m(t,s)ds|\ge\varepsilon\right)\le n\frac{E\left(\sum_{i=1}^n|\varepsilon_i\int_{A_i}E_m(t,s)ds|\right)^r}{n^{-r/12}} \\ &\le Cn\frac{(n^{-1}2^m)^r n^{r/2}}{n^{-1/12}}=Cn^{1-r/12}\to 0, \end{aligned}\tag{4.8}$$

therefore $|I_{n32}|=O(n^{-1/12})$. Therefore by (4.7) and (4.8) we have $I_{n3}\to 0\ \ (n\to\infty)$, combining (4.5) Theorem 2.2 is verified.

Proof of Theorem 2.3 We observe that

$$|g_n(t) - g(t)| \le |g_n(t) - Eg_n(t)| + |Eg_n(t) - g(t)|. \tag{4.9}$$

By Lemma 3.2 (I) we have $|Eg_n(t) - g(t)| \to 0$. Then according to (4.11) we only need to verify that

$$|g_n(t) - Eg_n(t)| \to 0. \tag{4.10}$$

By the Markov inequality and Lemma 3.4, Lemma 3.1 (III) we have

$$\begin{aligned} |Eg_n(t) - g_n(t)| &= \left|\sum_{i=1}^n \varepsilon_i \int_{A_i} E_m(t,s)ds\right|, \\ P\left(\sum_{i=1}^n \varepsilon_i \int_{A_i} E_m(t,s)ds > \varepsilon\right) &\le \frac{E\left|\sum\limits_{i=1}^n \int_{A_i} E_m(t,s)ds\varepsilon_i\right|^r}{\varepsilon^r} \\ &\le C\left|\int_{A_i} E_m(t,s)ds\right|^r n^{r/2} = Cn^{-1/6}. \end{aligned}$$

Therefore

$$\sum_n P\left(\sum_{i=1}^n \varepsilon_i \int_{A_i} E_m(t,s)ds > \varepsilon\right) < \infty,$$

then by Borel-Cantelli lemma, we have

$$\sum_{i=1}^n \varepsilon_i \int_{A_i} E_m(t,s)ds \to 0 \ \text{ a.s..} \tag{4.11}$$

Therefore by (4.10), (4.11) combining (4.9) and Theorem 2.3 is verified.

Proof of Theorem 2.4 Let

$$\sigma_n^2(x) = \mathrm{Var}(g_n(x)),\ S_n(t) = \sigma_n^{-1}(t)\{g_n(t) - g(t)\},\ Z_{ni} = \sigma_n^{-1}\varepsilon_i \int_{A_i} E_m(t,s)ds$$

for $i = 1, \cdots, n$, so that $S_n = \sum\limits_{i=1}^n Z_{ni}$. Let $k = [n/(p+q)]$. Then S_n may be split as $S_n = S_n' + S_n'' + S_n'''$, where

$$\begin{aligned} &S_n' = \sum_{m=1}^k y_{nm}, \quad S_n'' = \sum_{m=1}^k y'_{nm}, \quad S_n''' = y'_{nk+1}, \\ &y_{nm} = \sum_{i=k_m}^{k_m+p-1} Z_{ni}, \quad y'_{nm} = \sum_{i=l_m}^{l_m+q-1} Z_{ni}, \quad y'_{nk+1} = \sum_{i=k(p+q)+1}^{n} Z_{ni}, \\ &k_m = (m-1)(p+q)+1, \quad l_m = (m-1)(p+q)+p+1, \quad m = 1, \cdots, k. \end{aligned}$$

Thus, to prove the theorem, it suffices to show that

$$E(S_n'')^2 \to 0, \quad E(S_n''')^2 \to 0, \tag{4.12}$$

$$S_n' \xrightarrow{d} N(0,1). \tag{4.13}$$

By Lemma 3.4, (4.3), A5 and A6, we have

$$\begin{aligned}
E(S_n'')^2 &= \sigma_n^{-2}\sum_{m=1}^{k}\sum_{i=l_m}^{l_m+q-1}\left(\int_{A_i}E_m(t,s)ds\right)^2 E(\varepsilon_i)^2 \\
&\quad +2\sigma_n^{-2}\sum_{m=1}^{k}\sum_{l_m\le i\le j\le l_m+q-1}\int_{A_{i_1}}E_m(t,s)ds\int_{A_{i_2}}E_m(t,s)ds\mathrm{Cov}(\varepsilon_{i_1},\varepsilon_{i_2}) \\
&\quad +2\sigma_n^{-2}\sum_{1\le m\le s\le k}\sum_{i_1=l_m}^{l_m+q-1}\sum_{i_2=l_s}^{l_s+q-1}\int_{A_{i_1}}E_m(t,s)ds\int_{A_{i_2}}E_m(t,s)ds\mathrm{Cov}(\varepsilon_{i_1},\varepsilon_{i_2}) \\
&\le C\left\{kq+\sum_{m=1}^{k}\sum_{i=1}^{q-1}(q-i)\mathrm{Cov}(\varepsilon_1,\varepsilon_{i+1})+\sum_{m=1}^{k}\sum_{i_1=l_m}^{l_m+q-1}\sum_{s=m+1}^{k}\sum_{i_2=l_s}^{l_s+q-1}\mathrm{Cov}(\varepsilon_1,\varepsilon_{i+1})\right\}/n \\
&\le C\{kq+ku(1)+ku(p)\}/n\le Ckq\le Cqp^{-1}\to 0, \\
E(S_n''')^2 &\le \sigma_n^{-2}\sum_{i=k(p+q)+1}^{n}(\int_{A_i}E_m(t,s)ds)^2E(\varepsilon_i)^2 \\
&\quad +2\sigma_n^{-2}\sum_{k(p+q)+1\le i_1\le i_2\le n}\int_{A_{i_1}}E_m(t,s)ds\int_{A_{i_2}}E_m(t,s)ds\mathrm{Cov}(\varepsilon_{i_1},\varepsilon_{i_2}) \\
&\le C\left\{(n-k(p+q))+\sum_{i=1}^{n-k(p+q)-1}\mathrm{Cov}(\varepsilon_1,\varepsilon_{i+1})\right\}/n \\
&\le C\{(n/(p+q)-k)(p+q)+u(1)\}/n\le Cpn^{-1}\to 0.
\end{aligned}$$

Thus (4.12) holds.

We now proceed with the proof of (4.13). Let $\Gamma_n=\sum\limits_{1\le i<j\le k}\mathrm{Cov}(y_{ni},y_{nj})$, and $s_n^2=\sum\limits_{m=1}^{k}\mathrm{Var}(y_{nm})$, then $s_n^2=E(S_n')^2-2\Gamma_n$. Apply relation (4.12) to obtain $E(S_n')^2\to 1$. This would also imply that $s_n^2\to 1$, provided we show that $\Gamma_n\to 0$

Indeed, by assumption (A5) we obtain $u(q)\to 0$. Then by stationarity and theorem condition, it can be shown that

$$\begin{aligned}
|\Gamma_n| &\le \sum_{1\le i<j\le k}\sum_{s=k_i}^{k_i+p-1}\sum_{t=k_j}^{k_j+p-1}\sigma_n^{-2}\left|\int_{A_s}E_m(t,s)ds\int_{A_t}E_m(t,s)ds\right||\mathrm{Cov}(\varepsilon_s,\varepsilon_t)| \\
&\le C\sum_{i=1}^{k-1}\sum_{s=k_i}^{k_i+p-1}\left|\int_{A_s}E_m(t,s)ds\right|\sum_{j=i+1}^{k}\sum_{t=k_j}^{k_j+p-1}|\mathrm{Cov}(\varepsilon_{ns},\varepsilon_{nt})| \\
&\le C\sum_{i=1}^{k-1}\sum_{s=k_i}^{k_i+p-1}\left|\int_{A_s}E_m(t,s)ds\right|\cdot\sup_{j\ge 1}\sum_{t:|t-j|\ge q}|\mathrm{Cov}(\varepsilon_j,\varepsilon_t)| \\
&\le Cu(q)\to 0. \qquad (4.14)
\end{aligned}$$

Next, in order to establish asymptotic normality, we assume that $\{\eta_{nm}:m=1,\cdots,k\}$ are independent random variables, and the distribution of η_{nm} is the same as that η_{nm} for

$m=1,\cdots,k$. Then $E\eta_{nm}=0$ and $\mathrm{Var}(\eta_{nm})=\mathrm{Var}(y_{nm})$. Let $T_{nm}=\eta_{nm}/s_n, m=1,\cdots,k$, then $\{T_{nm}, m=1,\cdots,k\}$ are independent random variables with $ET_{nm}=0$ and $\mathrm{Var}(T_{nm})=1$. Let $\phi_X(t)$ be the characteristic function of X, then

$$
\begin{aligned}
&\left|\phi_{\sum\limits_{m=1}^{k} y_{nm}}(t)-e^{-t^2/2}\right| \\
\leq &\left|E\exp\left(it\sum_{m=1}^{k} y_{nm}\right)-\prod_{m=1}^{k} E\exp(ity_{nm})\right|+\left|\prod_{m=1}^{k} E\exp(ity_{nm})-e^{-t^2/2}\right| \\
\leq &\left|E\exp\left(it\sum_{m=1}^{k} y_{nm}\right)-\prod_{m=1}^{k} E\exp(ity_{nm})\right|+\left|\prod_{m=1}^{k} E\exp(it\eta_{nm})-e^{-t^2/2}\right|.
\end{aligned}
$$

By Lemmas 3.5, relation (4.14), we obtain that

$$
\begin{aligned}
&\left|E\exp\left(it\sum_{m=1}^{k} y_{nm}\right)-\prod_{m=1}^{k} E\exp(ity_{nm})\right| \\
\leq\ & 4t^2 \sum_{1\leq i<j\leq k}\ \sum_{s=k_i}^{k_i+p-1}\ \sum_{t=k_j}^{k_j+p-1} |\mathrm{Cov}(Z_s,Z_t)| \\
=\ & 4t^2 \sum_{1\leq i<j\leq k}\ \sum_{s=k_i}^{k_i+p-1}\ \sum_{t=k_j}^{k_j+p-1} \sigma_n^{-2}\left|\int_{A_s} E_m(t,s)ds\int_{A_t} E_m(t,s)ds\right| |\mathrm{Cov}(\varepsilon_s,\varepsilon_t)| \\
\leq\ & Ct^2u(q)\to 0.
\end{aligned}
$$

Thus, it suffices to show that $\eta_{nm}\xrightarrow{d} N(0,1)$ which, on account of $s_n^2\to 1$, will follow from the convergence $\sum\limits_{m=1}^{k} T_{nm}\xrightarrow{d} N(0,1)$. By the Lyapunov condition, it suffices to show that for some $r>2$,

$$
\frac{1}{s_n^r}\sum_{m=1}^{k} E|\eta_{nm}|^r\to 0. \tag{4.15}
$$

Using Lemma 3.4 and (A6), we have

$$
\begin{aligned}
\sum_{m=1}^{k} E|\eta_{nm}|^r &= \sum_{m=1}^{k} E|y_{nm}|^r=\sigma_n^{-r}\sum_{m=1}^{k} E\left|\sum_{i=k_m}^{k_m+p-1}\varepsilon_i\int_{A_i} E_m(t,s)ds\right|^r \\
&\leq C\sigma_n^{-r}\sum_{m=1}^{k}(2^m/n)^r p^{r/2}=C(pn^{-1})^{r/2-1}\to 0,
\end{aligned}
$$

so (4.15) holds. Thus the proof is completed.

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PA误差下回归函数小波估计的渐近性质

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摘 要: 本文研究了回归函数小波估计的渐进性质的问题. 利用概率不等式方法, 获得了函数$g(\cdot)$的小波估计量的r -阶矩相合, 依概率收敛和强收敛以及渐近正态性的结果, 所获的结果推广了其他混合相依下的相应结果.

关键词: 小波估计; 正相协; 相合性; 渐近正态性

MR(2010)**主题分类号**: 60F15 **中图分类号**: O211.4