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H_∞ Suboptimal Control for a Class of Singular Systems with Time-Delay: an LMI Approach *

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Abstract: The problem of H_∞ suboptimal control for a class of singular systems with time-delay is addressed in this paper. Under some conditions, a singular system with time-delay can be transformed into a system that consists of a differential equation and an algebraic equation. By using LMI approach, a memory-less controller is designed to solve the H_∞ suboptimal control problem for this class of systems.

Key words: singular system; time-delay; H_∞ suboptimal control; LMI approach

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一类带有时滞的广义系统的 H_∞ 控制：一种 LMI 方法

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摘要：利用线性矩阵不等式方法研究了一类带有时滞的广义系统的 H_∞ 控制问题. 在一定条件下, 一个时滞奇异系统可以转化成由一个微分方程和一个代数方程组成的系统, 基于线性矩阵不等式方法给出了使这类系统 H_∞ 干扰抑制性能指标满足要求的无记忆状态反馈控制设计.

关键词：广义系统；时滞；H_∞ 次优控制；LMI 方法

1 Introduction

Since the time delay is frequently viewed as a source of instability and encountered in various engineering systems such as chemical processes, long transmission lines in pneumatic systems, etc., the study of delay systems has received much attention and various topics have been discussed over the past years; see [1 ~ 4]. Meanwhile, singular system has also received much attention due to its profound and practical background. Many research results on regular systems are extended to singular systems[5 ~ 9]. Because of its nice numerical feature, the linear matrix inequality (LMI) are popularly used in the system and control theory[9,10]. The stabilizing problem of the singular systems is addressed by using LMI in [5]. The H_∞ control problem of the singular systems is discussed in [9]. Consider the following system:

$$E\dot{x} = Ax + Bx(t-d) + Cu$$

with E being singular matrix. The pencil $sE - A$ is proper and the pair (E, A) has no impulsion. The triple (E, A, C) is impulsively controllable. Under some conditions, this system can be transformed as

$$\begin{cases} \dot{x}_1(t) = A_3x_1 + A_1x_1(t-d) + A_2x_2(t-d) + E_1\omega + C_1u, \\ x_2(t) = B_1x_1(t-d) + B_2x_2(t-d) + E_2\omega + C_2u, \\ z = D_1x_1 + D_2x_2 + E_3\omega, \end{cases}$$

where $A_3, E_3, A_i, B_i, C_i, D_i, E_i (i = 1,2)$ are known constant matrices, x_1, x_2 denote the state of the system, $d > 0$ is a constant denoting the time delay, u, z, ω are the input, controlled output and disturbance respectively. In this paper we consider the case that the slow varying subsystem is free of disturbance (that is $E_1 = 0$),

$$\begin{cases} \dot{x}_1(t) = A_3x_1 + A_1x_1(t-d) + A_2x_2(t-d) + C_1u, \\ x_2(t) = B_1x_1(t-d) + B_2x_2(t-d) + E_2\omega + C_2u, \\ z = D_1x_1 + D_2x_2 + E_3\omega. \end{cases} \tag{1}$$

We assume that the solution of the system is the one

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with admissible initial condition. Throughout the paper, we simplify our notation of $x_1(t)$ to x_1 and simplify the other variables in the same way.

The objective of the controller design in this paper is as follows:

Definition 1[11] Consider system (1). For any given number $\gamma > \gamma_0 \geqslant 0$, where γ_0 is a non-negative number, find a state feedback control law so that the closed loop system is stable and the L_2-gain from the exogenous input to the regulated output is less than or equal to γ. This problem is called the H_∞ control problem with performance level γ.

Under some assumptions, we will construct state feedback control laws to solve the H_∞ control problem by using LMI.

2 Some results for unforced systems

Consider the following system

$$\begin{cases} \dot{x}_1(t) = A_0 x_1 + A_1 x_1(t-d) + A_2 x_2(t-d), \\ x_2(t) = B_0 x_1 + B_1 x_1(t-d) + B_2 x_2(t-d) + E_2\omega, \\ z = D_1 x_1 + D_2 x_2 + E_3\omega. \end{cases} \tag{2}$$

Let

$$\Delta = E_3^{\mathrm{T}} E_3 + E_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}) E_2 + E_3^{\mathrm{T}} D_2 E_2 + E_2^{\mathrm{T}} D_2^{\mathrm{T}} E_3, \tag{3}$$

$$\Lambda_1 = \begin{bmatrix} B_1^{\mathrm{T}}(Q_2+D_2^{\mathrm{T}}D_2)B_1 - Q_1 & B_1^{\mathrm{T}}(Q_2+D_2^{\mathrm{T}}D_2)B_2 \\ B_2^{\mathrm{T}}(Q_2+D_2^{\mathrm{T}}D_2)B_1 & B_2^{\mathrm{T}}(Q_2+D_2^{\mathrm{T}}D_2)B_2 - Q_2 \end{bmatrix}, \tag{4}$$

$$\Lambda_{21} = \begin{bmatrix} B_1^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)E_2 + B_1^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \\ B_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)E_2 + B_2^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \end{bmatrix}, \tag{5}$$

$$\Lambda_2 = \Lambda_{21}\Lambda_{21}^{\mathrm{T}}, \tag{6}$$

$$\rho = \lambda_{\max}(\Delta), \tag{7}$$

where Q_1 and Q_2 are definite positive matrices which meet some requirements.

It follows from (3) that $\Delta \geqslant 0$. Noting (7), we have $\rho \geqslant 0$. Let $\gamma_0 = \sqrt{\rho}$.

Assumption 1 In system (2), for any $\gamma > \gamma_0 \geqslant 0$, assume that $\Lambda < 0$, where

$$\Lambda = \Lambda_1 + \delta\Lambda_2,\ \delta\sigma = 1,\ \sigma = \gamma^2 - \rho. \tag{8}$$

Let

$$\begin{cases} \Omega_{11} = \begin{bmatrix} PA_0 + A_0^{\mathrm{T}}P + Q_1 + D_1^{\mathrm{T}}D_1 & PA_1 & PA_2 \\ A_1^{\mathrm{T}}P & -Q_1 & 0 \\ A_2^{\mathrm{T}}P & 0 & -Q_2 \end{bmatrix}, \\ \Omega_{12} = \begin{bmatrix} B_0^{\mathrm{T}}D_2^{\mathrm{T}}D_1 + D_1^{\mathrm{T}}D_2B_0 & D_1^{\mathrm{T}}D_2B_1 & D_1^{\mathrm{T}}D_2B_2 \\ B_1^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 \\ B_2^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 \end{bmatrix}, \end{cases} \tag{9a}$$

$$\Omega_{13} = \begin{bmatrix} B_0^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_0 & B_0^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_1 & B_0^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_2 \\ B_1^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_0 & B_1^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_1 & B_1^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_2 \\ B_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_0 & B_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_1 & B_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)B_2 \end{bmatrix},\ \Omega_1 = \sum_{i=1}^{3}\Omega_{1i}, \tag{9b}$$

$$\begin{cases} \Omega_{21} = \begin{bmatrix} B_0^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)E_2 + B_0^{\mathrm{T}}D_2^{\mathrm{T}}E_3 + D_1^{\mathrm{T}}D_2E_2 + D_1^{\mathrm{T}}E_3 \\ B_1^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)E + B_1^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \\ B_2^{\mathrm{T}}(Q_2 + D_2^{\mathrm{T}}D_2)E_1 + B_2^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \end{bmatrix}^{\mathrm{T}}, \\ \Omega_2 = \Omega_{21}^{\mathrm{T}}\Omega_{21}, \end{cases} \tag{10}$$

$$\Omega(P) = \Omega_1 + \delta\Omega_2. \tag{11}$$

We have the following result.

Theorem 1 Consider system (2) under Assumption 1. If for any $\gamma > \gamma_0 \geqslant 0$ the following LMI with $\Omega(P)$ being defined by (9), (10) and (11)

$$\Omega(P) < 0 \tag{12}$$

has positive definite solutions P, then system (2) has H_∞ performance with level γ.

Proof We introduce a Lyapunov Krasovskii function for system (2) of the form

$$V = x_1^{\mathrm{T}}Px_1 + \int_{t-d}^{t}[x_1^{\mathrm{T}}(s)Q_1x_1(s) + x_2^{\mathrm{T}}(s)Q_2x_2(s)]d_s. \tag{13}$$

The time derivative of V along the solution of the system (2) is given by

$$\dot{V} = 2x_1^{\mathrm{T}}P\dot{x}_1 + x_1^{\mathrm{T}}Q_1x_1 - x_1^{\mathrm{T}}(t-d)Q_1x_1(t-d) - x_2^{\mathrm{T}}(t-d)Q_2x_2(t-d) + x_2^{\mathrm{T}}(t)Q_2x_2(t). \tag{14}$$

Let

$$x = [x_1^{\mathrm{T}}, x_1^{\mathrm{T}}(t-d), x_2^{\mathrm{T}}(t-d), \omega^{\mathrm{T}}]^{\mathrm{T}}. \tag{15}$$

With the view of (2), (14) and (15), we have

$$\dot{V} = x^{\mathrm{T}}\begin{bmatrix} PA_0 + A_0^{\mathrm{T}}P + Q_1 & PA_1 & PA_2 & 0 \\ A_1^{\mathrm{T}}P & -Q_1 & 0 & 0 \\ A_2^{\mathrm{T}}P_0 & 0 & -Q_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}x +$$

$$x^{\mathrm{T}}\begin{bmatrix} B_0^{\mathrm{T}} \\ B_1^{\mathrm{T}} \\ B_2^{\mathrm{T}} \\ E_2^{\mathrm{T}} \end{bmatrix} Q_2[B_0 \quad B_1 \quad B_2 \quad E_2]x. \tag{16}$$

Letting

$$M_1 = \begin{bmatrix} PA_0 + A_0^{\mathrm{T}}P + Q_1 & PA_1 & PA_2 & 0 \\ A_1^{\mathrm{T}}P & -Q_1 & 0 & 0 \\ A_2^{\mathrm{T}}P & 0 & -Q_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{17}$$

(16) becomes

$$\dot{V} \doteq x^{\mathrm{T}}M_1x + x^{\mathrm{T}}\begin{bmatrix} B_0^{\mathrm{T}} \\ B_1^{\mathrm{T}} \\ B_2^{\mathrm{T}} \\ E_2^{\mathrm{T}} \end{bmatrix} Q_2[B_0 \quad B_1 \quad B_2 \quad E_2]x. \tag{18}$$

Note that

$$[(D_1x_1+E_3\omega)+D_2x_2]^{\mathrm{T}}[(D_1x_1+E_3\omega)+D_2x_2] = (D_1x_1+E_3\omega)^{\mathrm{T}}(D_1x_1+E_3\omega) + x_2^{\mathrm{T}}D_2^{\mathrm{T}}D_2x_2 + 2x_2^{\mathrm{T}}D_2^{\mathrm{T}}(D_1x_1 + E_3\omega). \tag{19}$$

Let

$$M_2 = \begin{bmatrix} D_1^{\mathrm{T}} \\ 0 \\ 0 \\ E_3^{\mathrm{T}} \end{bmatrix}[D_1 \quad 0 \quad 0 \quad E_3] = \begin{bmatrix} D_1^{\mathrm{T}}D_1 & 0 & 0 & D_1^{\mathrm{T}}E_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ E_3^{\mathrm{T}}D_1 & 0 & 0 & E_3^{\mathrm{T}}E_3 \end{bmatrix}, \tag{20}$$

$$\bar{M}_3 = \begin{bmatrix} B_0^{\mathrm{T}} \\ B_1^{\mathrm{T}} \\ B_2^{\mathrm{T}} \\ E_2^{\mathrm{T}} \end{bmatrix} 2D_2^{\mathrm{T}}[D_1 \quad 0 \quad 0 \quad E_3]. \tag{21}$$

It is clear that

$$\bar{M}_3 = \begin{bmatrix} 2B_0^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & 2B_0^{\mathrm{T}}D_0^{\mathrm{T}}E_3 \\ 2B_1^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & 2B_1^{\mathrm{T}}D_0^{\mathrm{T}}E_3 \\ 2B_2^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & 2B_2^{\mathrm{T}}D_0^{\mathrm{T}}E_3 \\ 2E_2^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & 2E_2^{\mathrm{T}}D_0^{\mathrm{T}}E_3 \end{bmatrix}. \tag{22}$$

Let

$$M_3 = \frac{1}{2}(\bar{M}_3 + \bar{M}_3^{\mathrm{T}}). \tag{23}$$

One gets

$$M_3 = \begin{bmatrix} B_0^{\mathrm{T}}D_2^{\mathrm{T}}D_1 + D_1^{\mathrm{T}}D_2B_0 & D_1^{\mathrm{T}}D_2B_1 & D_1^{\mathrm{T}}D_2B_2 & B_0^{\mathrm{T}}D_2^{\mathrm{T}}E_3 + D_1^{\mathrm{T}}D_2E_2 \\ B_1^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & B_1^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \\ B_2^{\mathrm{T}}D_2^{\mathrm{T}}D_1 & 0 & 0 & B_2^{\mathrm{T}}D_2^{\mathrm{T}}E_3 \\ E_2^{\mathrm{T}}D_2^{\mathrm{T}}D_1 + E_3^{\mathrm{T}}D_2B_0 & E_3^{\mathrm{T}}D_2B_1 & E_3^{\mathrm{T}}D_2B_2 & E_2^{\mathrm{T}}D_2^{\mathrm{T}}E_3 + E_3^{\mathrm{T}}D_2E_2 \end{bmatrix}. \tag{24}$$

n view of (20) and (24), (19) becomes

$$[(D_1x_1+E_3\omega)+D_2x_2]^{\mathrm{T}}[(D_1x_1+E_3\omega) + D_2x_2] = x^{\mathrm{T}}(M_2 + M_3)x + x_2^{\mathrm{T}}D_2^{\mathrm{T}}D_2x_2. \tag{25}$$

et

$$M_4 = \begin{bmatrix} B_0^{\mathrm{T}} \\ B_1^{\mathrm{T}} \\ B_2^{\mathrm{T}} \\ E_2^{\mathrm{T}} \end{bmatrix}(Q_2 + D_2^{\mathrm{T}}D_2)[B_0 \quad B_1 \quad B_2 \quad E_2],$$

$$M = \sum_{i=1}^{4} M_i. \tag{26}$$

he right-hand side (RHS) of (18) can then be e- alently written as follows, by adding the identically function $-\|z\|^2 + x^{\mathrm{T}}(M_2 + M_3)x + x_2^{\mathrm{T}}D_2^{\mathrm{T}}D_2x_2$ using (2):

$$\dot{V} = -\|z\|^2 + x^{\mathrm{T}}Mx. \tag{27}$$

Let

$$\chi = [x_1^{\mathrm{T}}, x_1^{\mathrm{T}}(t-d), x_2^{\mathrm{T}}(t-d)]^{\mathrm{T}}. \tag{28}$$

Recalling (15), we get $x = [\chi^{\mathrm{T}}, \omega^{\mathrm{T}}]^{\mathrm{T}}$. It is clear that $M = M^{\mathrm{T}}$. The matrix M can be partitioned as follows

$$M = \begin{bmatrix} \Omega_1 & \Omega_{21}^{\mathrm{T}} \\ \Omega_{21} & \Delta \end{bmatrix}, \tag{29}$$

where Δ, Ω_1 and Ω_{21} are defined by (4), (9b) and (10).

It follows from (27) ~ (29) that

$$\dot{V} = -\|z\|^2 + \chi^{\mathrm{T}}\Omega_1\chi + 2\chi^{\mathrm{T}}\Omega_{21}^{\mathrm{T}}\omega + \omega^{\mathrm{T}}\Delta\omega. \tag{30}$$

Recalling Assumption 2 and using (8) and (30), we have

$$\dot{V} \leqslant -\|z\|^2 + \gamma^2\|\omega\|^2 + \chi^{\mathrm{T}}\Omega_1\chi + 2\chi^{\mathrm{T}}\Omega_{21}^{\mathrm{T}}\omega - \sigma\|\omega\|^2. \tag{31}$$

By completing the square, one gets

$$\dot{V} \leqslant - \| z \|^2 + \gamma^2 \| \omega \|^2 + \chi^{\mathrm{T}}(\Omega_1 + \delta\Omega_{21}^{\mathrm{T}}\Omega_{21})\chi - \sigma\left\| \omega - \frac{1}{\sigma}\Omega_{21}\chi \right\|^2. \quad (32)$$

Let

$$\hat{\omega} = \omega - \frac{1}{\sigma}\Omega_{21}\chi. \quad (33)$$

It follows from (10), (11), (32) and (33) that

$$\dot{V} \leqslant - \| z \|^2 + \gamma^2 \| \omega \|^2 + \chi^{\mathrm{T}}\Omega(P)\chi - \sigma \| \hat{\omega} \|^2. \quad (34)$$

By using (12), we have

$$\dot{V} \leqslant - \| z \|^2 + \gamma^2 \| \omega \|^2. \quad (35)$$

The result is established by using (35).

Remark 1 Assumption 1 is a necessary condition for the existence of the solutions for LMI (12). In fact, Ω_1 and Ω_2 can be partitioned as $\Omega_1 = \begin{bmatrix} * & * \\ * & \Lambda_1 \end{bmatrix}$ and $\Omega_2 = \begin{bmatrix} * & * \\ * & \Lambda_2 \end{bmatrix}$. Thus $\Omega(P)$ can be partitioned as $\Omega(P) = \begin{bmatrix} * & * \\ * & \Lambda \end{bmatrix}$. It is clear that $\Lambda < 0$ if (12) holds. In the next section, we will consider the H$_\infty$ control problem for system (1).

3 Main results

We make the following assumption for system (1).

Assumption 2

$$C_2 \neq 0. \quad (36)$$

Let

$$\Theta_1 = Q_2 + D_2^{\mathrm{T}}D_2 + (Q_2E_2 + D_2^{\mathrm{T}}E_3 + D_2^{\mathrm{T}}D_2E_2)(Q_2E_2 + D_2^{\mathrm{T}}E_3 + D_2^{\mathrm{T}}D_2E_2)^{\mathrm{T}}. \quad (37)$$

It is clear that $\Theta_1 > 0$.

Let

$$\Theta = \Theta_1^{\frac{1}{2}} C_2. \quad (38)$$

It follows from Assumption 2 that $\Theta \neq 0$.

We first introduce a lemma.

Lemma 1 For any matrix which is not zero, there exists a matrix Σ_1 of full row rank such that

$$\Theta^{\mathrm{T}}\Theta = \Sigma_1^{\mathrm{T}}\Sigma_1. \quad (39)$$

Proof Noting that $\Theta^{\mathrm{T}}\Theta \geqslant 0$, then there exists an orthogonal matrix such that

$$\Theta^{\mathrm{T}}\Theta = N^{\mathrm{T}}\begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}N, \quad (40)$$

where $\Sigma > 0$.

Let

$$\Sigma_1 = \begin{bmatrix} \Sigma^{\frac{1}{2}} & \cdots & 0 \end{bmatrix} N. \quad (41)$$

We see that Σ_1 is of full row rank and satisfies (39). The proof ends.

It follows from Σ_1 being of full row rank that $\Sigma_1\Sigma_1^{\mathrm{T}}$ is nonsingular. We can define a matrix Π as

$$\Pi = \Sigma_1^{\mathrm{T}}(\Sigma_1\Sigma_1^{\mathrm{T}})^{-2}\Sigma_1. \quad (42)$$

Let

$$A_0 = A_3 - 2\sigma C_1\Pi C_1^{\mathrm{T}}P, \ B_0 = -2\sigma C_2\Pi C_1^{\mathrm{T}}P. \quad (43)$$

We have the following theorem.

Theorem 2 Consider system (1) under Assumptions 1,2. If for any $\gamma > \gamma_0 \geqslant 0$ the LMI $\Omega(P) < 0$ and with $\Omega(P)$ being defined by (9), (10) and (11), has positive definite solutions P, where A_0 and B_0 are defined by (42), then the H$_\infty$ control problem for system (1) is solved by the state control law

$$u = -2\sigma\Pi C_1^{\mathrm{T}}Px_1, \quad (44)$$

where σ is defined by (8) and Π defined by (37) ~ (39) and (42).

Proof We will make clear that $\Omega(P)$ is a linear matrix with respect to matrix P. To do this, we see that the nonlinear terms of P in $\Omega(P)$ are only in the first row and first column entry of $\Omega(P)$. The entry is denoted by $\Omega^{(11)}$. It follows from (9), (10), (11) and (38) that

$$\Omega^{(11)} = -4\sigma PC_1\Pi C_1^{\mathrm{T}}P + 4\sigma PC_1\Pi C_2^{\mathrm{T}}[Q_2 + D_2^{\mathrm{T}}D_2 + (Q_2E_2 + D_2^{\mathrm{T}}E_3 + D_2^{\mathrm{T}}D_2E_2)(Q_2E_2 + D_2^{\mathrm{T}}E_3 + D_2^{\mathrm{T}}D_2E_2)^{\mathrm{T}}]C_2\Pi C_1^{\mathrm{T}}P + \#, \quad (45)$$

where # denote the matrices are of less interest.

With the view of (37), (38) and (45), we have

$$\Omega^{(11)} = 4\sigma PC_1[\Pi\Theta^{\mathrm{T}}\Theta\Pi - \Pi]C_1^{\mathrm{T}}P + \#. \quad (46)$$

It follows from (39) and (42) that

$$\Pi\Theta^{\mathrm{T}}\Theta\Pi = \Pi\Sigma_1^{\mathrm{T}}\Sigma_1\Pi = \Sigma_1^{\mathrm{T}}(\Sigma_1\Sigma_1^{\mathrm{T}})^{-2}\Sigma_1\Sigma_1^{\mathrm{T}}\Sigma_1\Sigma_1^{\mathrm{T}}(\Sigma_1\Sigma_1^{\mathrm{T}})^{-2}\Sigma_1 = \Sigma_1^{\mathrm{T}}(\Sigma_1\Sigma_1^{\mathrm{T}})^{-2}\Sigma_1 = \Pi. \quad (47)$$

It is clear from (45), (46) that $\Omega(P)$ is a linear matrix with respect to matrix P. With the view of (1), (43) and (44), we see that the resulted closed-loop system of (1) with control law (44) is (2). The conclusion of Theorem 2 is established by using Theorem 1.

4 Conclusion

The problem of H_∞ suboptimal control for a class of singular systems with time-delay is investigated by using LMI approach. Under some conditions, a singular system with time-delay can be transformed into a system that consists of a differential equation and an algebraic equation. The H_∞ suboptimal control problem of this class of systems is solved by using LMI approach. All the results are obtained with the assumption that the differential subsystem (slow varying subsystem) is free of disturbance. The state feedback control law is only constructed by the state of the slow varying subsystem. The analysis results are sufficient and may not be necessary. The future work will be focused on the following:

· Analysis of the system with full state feedback;

· Control design for slow varying subsystem with disturbance;

· Output feedback control.

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